Optimal Choice Mistakes: Theory and Experiment

David Walker-Jones*

University of Toronto

david.walker.jones@mail.utoronto.ca

April 5, 2021

To view the latest version of my paper, please use the following link: Optimal Choice Mistakes. Or, visit my website: dwalkerjones.com.

Abstract

To accurately predict changes in behavior and estimate preferences it is crucial to understand how the cost of information determines optimal choice mistakes. I characterize the choice patterns that are consistent with two major models of costly learning when price changes and extend the analysis to aggregate data that incorporates heterogeneous decision makers. I test my theoretical results in a lab setting and find that the heterogeneity of subjects is key for understanding behavior, and that subjects are sophisticated when choosing both *when* and *what* to learn. Subjects are most likely to learn in the setting predicted by the theory, and there is evidence that they accumulate information gradually in order to alter the relative probabilities of different kinds of choice mistakes.

1 Introduction

Decision makers are increasingly inundated with information. Incorporating this information into optimal choices requires the costly investment of time and effort. As a result, decision makers often do not acquire all the information that may be relevant to their decision. Even if a decision maker chooses what to learn in an optimal way, lack of information may cause them to select an option with a relatively low payoff. If a decision maker does not choose the option with the highest

*I would like to thank Rahul Deb, Yoram Halevy, Johannes Hoelzemann, Ran Spiegler, and Colin Stewart, for their helpful advice and support.

payoff, then their choice can be deemed a mistake. This paper studies costly learning and the choice mistakes that result from optimal information acquisition both theoretically and experimentally.

Costly information acquisition is of consequence to economic models, and the inferences one can make from data, for two main reasons. First, costly information acquisition is significant for predicting what a decision maker would choose if, for example, prices change. If the information a decision maker acquires changes when the price of an option changes, knowing this is crucial for accurately predicting the probability of that option being selected.

Second, understanding the choice mistakes that are caused by costly information acquisition is significant for understanding the preferences of a decision maker. The revealed preference approach is predicated on the assumption that the option the decision maker selects is preferred to the options they do[1] not select. When partial information acquisition can result in the agent selecting an option that has a relatively low payoff, it makes it more difficult to recover preferences. To identify preferences, one needs a good understanding of the probabilities of different forms of optimal choice mistakes.

This paper focuses on a simple model in which the agent has two options, a safe option with a payoff that is normalized to zero, and an uncertain option that takes either a low or a high value. To select the uncertain option the agent must pay a price. Their payoff from selecting the uncertain option is thus its value minus the price. The agent does not know the value of the uncertain option but does know the price and the probability that the value of the uncertain option is high, which is referred to as their prior belief. The option that is best for the agent depends on both the price and the realized value of the uncertain option. If the price is higher than the agent's value for the uncertain option, then selecting the uncertain option is a mistake. If the price is lower than the value of the uncertain option, then not selecting the uncertain option is a mistake.

I take a revealed preference approach, which means that the main results do not require that preferences are known by the researcher. In this paper's model it is easy to show that when the agent learns they are more likely to select the uncertain option if it has a high value compared to if it has a low value. One can thus infer which value is low and which is high, and this is all that is required by my model for analysis. This is in contrast to what is typical in the literature on rational inattention. For instance, Caplin, Dean, and Leahy (2017) and Pomatto, Strack, and Tamuz (2019) assume that the researcher can observe the preferences of the agent.

[1] I use 'they' instead of she or he. When I use 'they' as a singular noun I conjugate the verb, i.e. I use 'do' instead of 'does,' as recommended by the APA style guide. (*Singular "They"*, n.d.)

I model choice mistakes as the result of the cost of acquiring information: it is not worth the agent's time and effort to learn to the extent that they always select the option with the highest value. There are many ways to model costly learning, and the predictions of major models are not yet fully understood, even for a basic comparative static in price.

In the literature on costly information acquisition it is standard to model the learning of the agent as a signal structure: the agent's learning results in them receiving a 'signal' that provides them with information. For the sake of this explanation, one can think of each signal as just being a real number the agent observes. The probability of different realizations of the signal depends on the realized value of the uncertain option. Each signal results in a posterior belief, which is the probability that the value of the uncertain option is high after the signal is received. The joint distribution between signals and the value of the uncertain option is the 'signal structure.'

To study costly information acquisition, one needs a 'cost function' for information, which is to say the researcher needs a function that assigns costs to different signal structures. There are two natural domains for a cost function for information.

In 'Uniformly Posterior Separable' (Caplin et al., 2017) models, the current standard in the field of rational inattention that follows from the work of Sims (2003), the domain of the cost function for information is the posterior beliefs that the agent reaches: each signal results in a posterior, and the agent pays for the posterior based on how 'informed' the posterior is according to some function. In this paper such a model is called an ex-post accuracy model,[2] because the agent pays for information based on how sure they are of their decision being correct ex-post. That is, when the decision maker chooses the uncertain option what is the probability that it is the better alternative? Similarly, when the decision maker chooses the safe option what is the probability that it is the better alternative?

Ex-post accuracy models face two main criticisms. One criticism is that if 'perfectly learning,' which is learning the realized value of the uncertain option with certainty, has a finite but strictly positive cost, then changing the prior belief of the agent can change the cost of perfectly learning.[3] This is unintuitive for some because they think of the agent as perfectly learning by conducting a 'task.' Say that the 'task' is reading the details on the back of a certain option's package, then it may be unintuitive for the cost of reading the details to depend on the prior belief of the agent. Another

[2]This paper actually studies a slightly more general set of cost functions compared to the Uniformly Posterior Separable functions studied by Caplin et al. (2017) because this paper only impose weak, not strict, convexity onto the cost function.

[3]This is true whenever not learning anything, and having a posterior that is the same as the prior with probability one, has a cost of zero.

criticism that ex-post models sometimes face is that they allow too much freedom in information choice, since they do not typically put any restrictions on the distribution over posteriors the agent can select.

In response to these criticisms of the ex-post model, it is natural to consider an alternate model in which the agent has less flexibility when choosing what information to acquire. Suppose the agent selects a simple signal structure with only two signal realizations, call them '0' and '1.' Further, suppose that the agent simply selects an 'accuracy,' which is the probability of receiving the '1' signal when the realized value of the uncertain option is high, and the '0' signal when it is low. In this paper such a model is called an ex-ante accuracy model, because the agent pays for information based on how sure they are of their decision being correct ex-ante. That is, when the value of the uncertain option is high what is the probability that it is selected? Similarly, when the value of the uncertain option is low what is the probability that the safe option is selected? The domain of the cost function in an ex-ante accuracy model is the set of accuracies the agent can select.

I study a simple but flexible model that is general enough to analyse the implications of a broader class of models, of which ex-post accuracy models and ex-ante accuracy models are strict subsets. The domain of the cost function for information I use to study costly learning is the observed behavior of the agent, which is described by the probability of them selecting the uncertain option when its value is low, and the probability of them selecting the uncertain option when its value is high. Because the signal structure of an agent is not observed, explicitly modelling it is not helpful for analysis. The observed behavior model subsumes binary signals as a special case, but allows for more generality that is useful, for instance, for discussing exogenous learning.

Using this model of observed behavior, I am the first to fully characterize the choice patterns that are consistent with price changes and models of both ex-ante and ex-post accuracy. I then discuss the implications of the models for datasets that aggregate choices across agents who might have heterogeneous prior beliefs and heterogeneous information costs, and characterize the observed behavior that is consistent with the resultant aggregate version of the ex-post model.

I show that if price increases the ex-ante model is characterized by choice behvaior such that, if the agent chooses to acquire information before and after the change, they are either more likely to make a mistake given both realized values of the uncertain option, or less likely to make a mistake given both realized values of the uncertain option. Which of these two possibilities occurs is determined by the agent's prior belief.

I also show that if price increases the ex-post model is characterized by choice behavior such that, if the agent chooses to acquire information before and after the change, they are less likely to have made a mistake after selecting the uncertain option, and more likely to have made a mistake after selecting the safe option. In this context, conditioning on their action, the agent's prior belief is essentially *irrelevant* to their probability of an ex-post mistake.

When I allow for heterogeneous prior beliefs and costs for information, the resultant aggregate version of the ex-post model predicts changes in behavior that are inconsistent with a representative agent version of the ex-post model. I show that if price increases such a model is characterized by choice behavior such that it is more likely there is a mistake if the value of the uncertain option is high and less likely there is a mistake when the value of the uncertain option is low.

I further show that an aggregate version of the ex-ante model imposes strictly less structure on behavior compared to the aggregate version of the ex-post model. Again, an aggregate version of the ex-ante model can produce behavior that is inconsistent with a representative agent version of the ex-ante model.

These results may be surprising to some readers whose intuition is based on random utility models. If one does not restrict the value distribution of agents to be in a particular parametric class when a random utility model is used, then aggregating choices over heterogeneous agents always produces behavior that is consistent with some representative agent.

Using a novel design, I evaluate the predictions of the theoretical models experimentally in a setting where the researcher can observe choice mistakes that should be attributed to the cost of acquiring information. In the experiment, subjects are presented with a sequence of decision problems in which they decide between a safe option and an uncertain option.

If a subject wants to learn about the value of the uncertain option in a decision problem they have to choose to 'get more information,' which results in 100 small dots appearing, each of which is either red or green. If the subject learns which color constitutes the majority of the small dots, they learn the value of the uncertain option. In each decision problem there are 49 small dots of one color of and 51 of the other. This design allows the researcher to observe whether a subject attempted to acquire information in each decision problem, which results in a better understanding of how accurately they learn when they do in fact learn, since there is no need to average choice mistakes over instances where subjects do and do not try to acquire information. This is helpful for a number of reasons. In particular, it allows me to reject the aggregate ex-ante model in favor of the aggregate ex-post model, which is not possible with a standard dataset on behavior since in a standard dataset the aggregate ex-ante model imposes strictly less structure compared to the aggregate ex-post model.

Both the ex-ante and ex-post accuracy models predict that, given a prior, a subject should have more incentive to learn if the price increases or decreases towards the mean value of the uncertain option. In the experiment a subject is more likely to choose to 'get more information' in the settings predicted by the theory, but there is a great deal of heterogeneity across subjects in terms of how willing they are to 'get more information.'

There is also evidence that if a subject does choose to 'get more information,' they accumulate information gradually in a way that allows them to achieve ex-ante accuracies that differ depending on the realized value of the uncertain option. When the realized value of the uncertain option does not align with the value that was most likely according to a subject's prior belief, they spend more time acquiring information. This suggests that a subject chooses to count the dots in a way they are aware could lead to errors, and then recounts if initial counts produce unexpected results. Because of this, subjects achieve ex-ante accuracies that differ across realizations of the value of the uncertain option in a statistically significant fashion, which would be more in line with the ex-post model. This is all true even though subjects could presumably learn as accurately as they want to with a single count of the small dots.

When price changes the ex-post model does better than the ex-ante model predicting how average choices change when choices are aggregated from different decision makers, but does poorly when both price and prior belief change.

The ex-ante model does poorly predicting changes in behavior when price changes because it seems to impose too much structure on learning, and misses the ways in which the probability of the subject choosing to 'get more information' can change when price changes. The model predicts there are only two prices where the agent is indifferent between choosing to 'get more information' and not doing so. Between these prices the agent strictly prefers to 'get more information,' and below or above these two prices the agent strictly prefers to not 'get more information.' Because agents are heterogeneous, however, the probability of choosing to 'get more information' can decrease without dropping to zero when price increases, as is observed in the data. As a result, choice data does not resmble the predictions of the ex-ante model if choices are aggregated from different subjects. This is one of the experimental results that indicates that the heterogeneity of agents is important for understanding aggregate behavior. In the experiment the identity of a subject is observed, but in many real world contexts data is aggregated over individuals, so it is important to understand how behavior can change in such an aggregate dataset.

The ex-post model does poorly predicting changes in behavior when price and prior belief both change because it predicts that if the agent is still willing to learn after their prior belief changes then (generically) their set of optimal ex-post accuracies does not change. To achieve the same ex-post accuracies when prior belief changes the agent would need to alter their relative probabilities of committing the two kinds of ex-ante mistakes in a very specific way. For instance, to achieve the same ex-post accuracies after a change in the agent's prior the agent might need to achieve a specific increase in their ex-ante accuracy when the value of the uncertain option is low and a specific decreases in their ex-ante accuracy when the value of the uncertain option is high, or vice versa. This might be difficult, and result in an unnecessarily high cost of learning, because the agent would need to alter their accuracy in a specific way based on the value of the uncertain option, but they do not know the value of the uncertain option unless they learn in an accurate fashion, and if they learn too accurately they may not achieve the same ex-post accuracies.

The data indicates that subjects either do not have enough flexibility selecting what information to acquire, or it is not optimal for them, to alter the relative probabilities of ex-ante choice mistakes so as to achieve the same ex-post accuracies when prior belief changes. If one wants an ex-post model to reflect this pattern from the data, then one needs to allow the agent's 'Posterior Separable' cost function to change when their prior changes, as is permitted by the aggregate version of the ex-post model.

The rest of the paper is organized as follows: Section 2 introduces a general costly learning model, and characterizes the behavior that is consistent with price changes and several models of costly learning. Section 3 introduces the experiment, and discusses the agent's decision to 'get more information' in the context of the experiment. Section 4 explores the results of the experiment, and introduces some theory results that are tailored to the specifics of the experiment. Section 5 is a literature review, and Section 6 concludes.

2 Model

Consider an agent choosing between two options, option X and option Y.[4] Option X is the **safe option** because the agent knows their payoff from selecting it is 0. Option Y is the **uncertain**

[4]Because the model allows for any finite number of prices, this model is equivalent to one where the agent has any finite number of options, and any finite number of prices, as long as there is only one option Y that is costly for the agent to learn the value of.

option because there is some uncertainty about the payoff the agent gets from selecting it. The agent's **value** for option Y, $u(\omega)$, is determined by the **state** $\omega \in \Omega = \{\underline{\omega}, \overline{\omega}\}$, with $u(\underline{\omega}) < u(\overline{\omega})$. The state ω is an obscure feature because it is not known by the agent. To select the uncertain option the agent must pay the **price** $p \in \mathbb{R}$. The price p is a prominent feature, which means it is known by the agent. The payoff the agent receives from selecting option Y is $u(\omega) - p$.

Two values are all that is relevant for the experiment, help us impose more structure, and ease exposition. Further, focusing on learning that can be represented by yes-or-no questions may be appealing given results in the psychology and psychophysics literature. See the literature review in Section 5 for more details.

The agent knows the distribution $\mu \in \Delta(\Omega)$ that describes the probability of option Y obtaining its different values, which is referred to as their **prior belief**. The belief μ is assumed to assign a strictly positive probability to both values unless it is otherwise stated.

Given μ, the **behavior** at a set of prices $\mathcal{P} \subseteq \mathbb{R}$, is a mapping $s : \mathcal{P} \to S \equiv [0, 1] \times [0, 1]$. At each price $p \in \mathcal{P}$ the researcher observes the **information outcome** $s(p) = (\underline{s}(p), \overline{s}(p)) = (\Pr(Y|\underline{\omega}, p), \Pr(Y|\overline{\omega}, p))$, that describes the probability of the agent selecting option Y at each ω, denoted $\Pr(\mathrm{Y}|\omega, p) \equiv 1 - \Pr(\mathrm{X}|\omega, p)$.

This paper's focus on learning *outcomes* instead of on *signals* differs from what is typical in the literature on costly learning. This paper's model focuses on outcomes because this is the observable behavior that is measured in a typical dataset. Modelling the agent as selecting outcomes instead of signals is without loss since the agent would otherwise simply select the cheapest signal structure that generates the desired probability of selecting option Y given each realization of ω. Or, one could model the agent as selecting a signal structure that always results in one of two signals, '0' or '1,' where the probability of receiving the '1' signal when the value is $u(\underline{\omega})$ is $\underline{s}$, and the probability of receiving the '1' signal when the value is $u(\overline{\omega})$ is $\overline{s}$.

The agent is said to **learn** at a price p if $\Pr(\mathrm{Y}|\underline{\omega}, p) \neq \Pr(\mathrm{Y}|\overline{\omega}, p)$ and $\mu(\overline{\omega}) \in (0, 1)$. In other words, the agent is learning whenever their probability of selecting option Y changes when the realization of ω changes, holding price and belief fixed, because this is indicative of the agent at least partially differentiating between $\underline{\omega}$ and $\overline{\omega}$.

How does one measure the accuracy of the agent's choices? Whenever the agent learns, their accuracy can be measured in either an ex-ante or ex-post fashion. Given the state ω and the price p, **ex-ante accuracy** measures the probability that the correct option is selected: $\Pr(\mathrm{Y}|\overline{\omega}, p)$ and $\Pr(\mathrm{X}|\underline{\omega}, p)$. Given the option selected by the agent, X or Y, the price, p, and the prior belief, μ,

ex-post accuracy measures the probability that the selected option is the best available:

$$\Pr(\overline{\omega}|\mathrm{Y},\, p) \equiv \frac{\Pr(\mathrm{Y}|\overline{\omega},\, p)\mu(\overline{\omega})}{\Pr(\mathrm{Y}|\overline{\omega},\, p)\mu(\overline{\omega}) + \Pr(\mathrm{Y}|\underline{\omega},\, p)\mu(\underline{\omega})},$$

and

$$\Pr(\underline{\omega}|\mathrm{X},\, p) \equiv \frac{\Pr(\mathrm{X}|\underline{\omega},\, p)\mu(\underline{\omega})}{\Pr(\mathrm{X}|\underline{\omega},\, p)\mu(\underline{\omega}) + \Pr(\mathrm{X}|\overline{\omega},\, p)\mu(\overline{\omega})}.$$

This paper studies costly information acquisition, which means the agent pays a cost for the accuracy of their choices. Due to a simple application of Bayes' Rule, measuring one of ex-ante and ex-post accuracy is equivalent to measuring the other, and it may seem redundant to consider both. But, one of the theoretical insights of this paper is that ex-ante and ex-post modelling strategies lead to contrasting comparative statics for how behavior changes when price changes. In both models it is natural to impose convexity on the cost function for accuracy so that 'more information' is weakly more costly. Surprisingly, which measures of accuracy the convex cost function is applied to changes behavior predictions, so in this paper both ex-ante and ex-post accuracy models are studied.

2.1 Costly Learning

This paper identifies the behavioral predictions associated with price changes. To do this, the model needs to impose structure on the 'cost function' for information, which is to say, one needs to make assumptions about how difficult it is for subjects to generate different accuracy outcomes. There are many ways to model costly information acquisition, however, and they generally produce different predictions for behavior, as is demonstrated in this section. This section begins with a very general model of costly information acquisition, and then moves on to more specific models.

Given the belief of the agent, μ, the **cost function** for information outcomes is a mapping from S onto the positive reals, $C_\mu : S \to \mathbb{R}_+$, which satisfies Assumption 1 and Assumption 2 below. The cost function C_μ describes the minimal cost of achieving each potential observed behavior. The cost function depends on μ because in some contexts it may be intuitive to let the cost of certain information outcomes change when the agent's belief changes. The cost function does not depend on price, which is to say this paper's model assumes that a price change does not impact the cost of the agent achieving a given information outcome. This does not mean that the cost of the information outcome the agent selects does not change when price changes, the realized cost $C_\mu(s(p))$ might change when p changes, but the function C_μ does not change if p changes.

What structure is natural to impose on C_μ? If the agent can achieve an outcome without doing any learning, then it is natural to think the outcome should be costless. So, the agent should be able to select option Y, or option X, or randomize over these options, all without incurring any learning costs. Further, it is natural to assume that the agent can randomize over information outcomes without cost. So, if a given information outcome is a convex combination of other information outcomes, then the cost of the information outcome should be weakly lower than the appropriate convex combination of the costs of the other information outcomes. This is equivalent to assuming that more information in a Blackwell (1953) sense is weakly more costly.

Assumption 1: the agent can choose option Y with the same probability for both realizations of ω without any learning cost, which is to say $\forall x \in [0,\, 1]$, $C_\mu(x,\, x) = 0$.

Assumption 2: the agent can randomize over information outcomes without cost, which is to say C_μ is a weakly convex function.

Behavior is rationalized by a costly learning model if there exist values of $\underline{\omega}$ and $\overline{\omega}$ and a cost function for information outcomes such that the behavior is optimal.

Definition: Given a prior belief μ and a set of prices $\mathcal{P}$, the behavior is **rationalized by a costly learning model** if there are values $u(\underline{\omega}) < u(\overline{\omega})$ and a cost function for information outcomes C_μ such that, $\forall\, p \in \mathcal{P}$,

$$s(p) = (\underline{s}(p),\, \overline{s}(p)) \in \underset{(\underline{s},\, \overline{s}) \in S}{\arg\max} \Big(\underline{s}\mu(\underline{\omega})(u(\underline{\omega}) - p) + \overline{s}\mu(\overline{\omega})(u(\overline{\omega}) - p) - C_\mu(\underline{s},\, \overline{s}) \Big).$$

Given the generality of the above definition of rationalization, one might think it does not impose any structure on the behavior of the agent. While quite flexible, the above definition does impose some structure. This is because when price increases, the payoff of some information outcomes decrease more quickly than others.

Given p and μ, the expected **payoff** the agent receives when they choose an information outcome $s = (\underline{s},\, \overline{s})$ is:

$$\underline{s}\mu(\underline{\omega})(u(\underline{\omega}) - p) + \overline{s}\mu(\overline{\omega})(u(\overline{\omega}) - p) - C_\mu(\underline{s},\, \overline{s}).$$

When p increases the payoff of option Y decreases. As a result, when p increases, the value of an information outcome $s = (\underline{s},\, \overline{s}) \in S$ decreases in proportion to the **unconditional probability** of choosing option Y that the information outcome results in, which is $\underline{s}\mu(\underline{\omega}) + \overline{s}\mu(\overline{\omega})$. So, information outcomes that create lower unconditional probabilities of selecting option Y decrease in value less quickly when p increases. This suggests the agent should select information outcomes that result in a lower unconditional probability of selecting option Y when price increases, as is shown in Proposition 1.

Proposition 1. Given a prior belief μ and a finite and non-empty set of prices $\mathcal{P}$, the behavior is rationalized by a costly learning model if and only if the following three properties are satisfied:
(i) The agent is weakly more likely to chose option Y when its value is high:

$$\forall\, p \in \mathcal{P}: \ \Pr(\mathrm{Y}|\underline{\omega},\, p) \leq \Pr(\mathrm{Y}|\overline{\omega},\, p).$$

(ii) The agent is weakly less likely to select option Y (unconditionally) when price increases:

$$\Pr(\mathrm{Y}|p) \equiv \mu(\overline{\omega})\Pr(\mathrm{Y}|\overline{\omega},\, p) + \mu(\underline{\omega})\Pr(\mathrm{Y}|\underline{\omega},\, p) \text{ is weakly decreasing in } p.$$

(iii) If there is a price at which the agent randomizes over selecting option X and option Y without learning, then they do not learn at any price, and select either option X or option Y with probability one at all other prices:

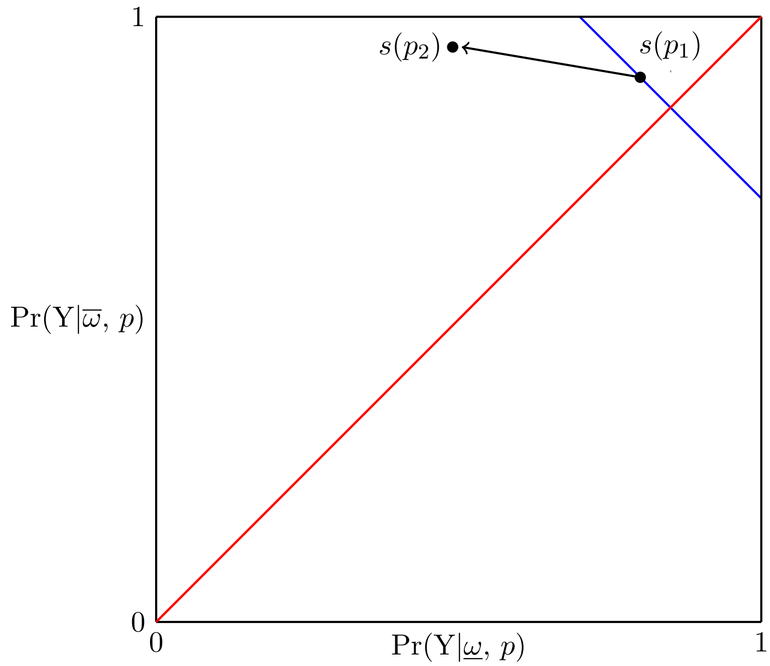
$$\text{if } \exists\, p \in \mathcal{P}: \ \Pr(\mathrm{Y}|\underline{\omega},\, p) = \Pr(\mathrm{Y}|\overline{\omega},\, p) \in (0,\, 1), \ \text{ then } \forall\, \tilde{p} \in \mathcal{P}\backslash p: \ \Pr(Y|\tilde{p}) \in \{0,\, 1\}.$$

Proof. See Appendix 2.

Figure 1 illustrates the implications of Proposition 1. Figure 1 depicts S, where the horizontal axis describes the probability of selecting option Y if the value is $u(\underline{\omega})$, $\Pr(\mathrm{Y}|\underline{\omega},\, p)$, the vertical axis describes the probability of selecting option Y if the value is $u(\overline{\omega})$, $\Pr(\mathrm{Y}|\overline{\omega},\, p)$, and the red line is the information outcomes that result in the same probability of selecting option Y given either realization of the value (these are the information outcomes Assumption 1 requires to have a cost of zero). In Proposition 1, part **(i)** requires that each $s(p)$ be on or above the red line. The level set of $s(p_1)$ is the collection of points that create the same unconditional probability of selecting option Y.[5] Part **(ii)** requires that $s(p_2)$ be on or below the level set of $s(p_1)$, which is depicted in blue, if $p_2 > p_1$. Part **(iii)** requires that $s(p)$ not be in the interior of S and on the red line unless for all other $\tilde{p} \in \mathcal{P}$ the behavior is such that $s(\tilde{p}) = (0,\, 0)$ or $s(\tilde{p}) = (1,\, 1)$.

[5]The blue line is defined by $\underline{s}\mu(\underline{\omega}) + \overline{s}\mu(\overline{\omega}) = \underline{s}(p_1)\mu(\underline{\omega}) + \overline{s}(p_1)\mu(\overline{\omega})$. The slope depicted in Figure 1 corresponds to $\mu(\underline{\omega}) = \mu(\overline{\omega}) = \frac{1}{2}$.

Figure 1: Implications of a general costly learning model ($p_1 < p_2$)



If I do not impose any additional structure on the learning costs of the agent, beyond Assumption 1 and Assumption 2, then Proposition 1 tells us the behavioral patterns that are consistent with this general model of costly learning. One may wish to know the least general model that can rationalize observed behavior, however, so that the strongest possible predictions can be made. The following subsections present several approaches for imposing natural restrictions on the cost function for information outcomes and identify the structure they impose on behavior.

2.2 Ex-ante Accuracy Model

If learning is costly for the agent, they may decide how much effort to committee to learning, but they may not have much flexibility deciding what to learn. Perhaps there is one task available to the agent for acquiring information, they can learn by calling a help line, or they can try to find a store clerk to talk to, or they can read about a product on-line. The agent can presumably decide how much effort to commit to this task, and more effort presumably results in more accurate information, but a one dimensional choice of effort may be appropriate for modelling the learning decisions of the agent. There are a number of papers that employ one-dimensional choices of information (e.g., Verrecchia, 1982; Colombo, Femminis, & Pavan, 2014). Of particular relevance, Verrecchia (1982) studies a model of a market with a riskless bond and a risky asset in which traders learn about the risky asset by selecting the variance of a normally distributed signal. Modelling the agent as choosing a learning strategy from a one-dimensional set is appealing because it would impose structure that helps strengthen predictions.

If the agent is only selecting the effort with which they learn, they may not have much control over the relative magnitudes of the two measures of ex-ante accuracy: more effort leads to higher ex-ante accuracies, but, given a fixed value for $\Pr(\mathrm{X}|\underline{\omega}, p)$, the agent does not alter the value of $\Pr(\mathrm{Y}|\overline{\omega}, p)$, or vice versa. In particular, when the agent does decide to acquire information, their learning may result in a symmetric ex-ante accuracy that is the same for either realization of ω: $\Pr(\mathrm{Y}|\overline{\omega}, p) = \Pr(\mathrm{X}|\underline{\omega}, p) = q \in (\frac{1}{2}, 1]$. What would this assumption mean in terms of the implicit signal structure of the agent? It would mean that the agent is receiving two signals, one that tells them that the realized value of option Y is high, one that tells them that the realized value of option Y is low, and the probability of receiving a correct signal is the same for either realized value of option Y.

Imposing the assumption that learning results in a symmetric ex-ante accuracy is a natural starting point due to the simplicity of the assumption. It also help us impose substantially more structure on predicted behavior compared to the more flexible ex-ante accuracy model in the next subsection.

Note that an ex-ante accuracy model of the form above can still result in behavior that does not have symmetric ex-ante accuracies: $\Pr(\mathrm{Y}|\overline{\omega}, p) \neq \Pr(\mathrm{X}|\underline{\omega}, p)$ for some p. This is possible because the agent may be randomizing over acquiring information and not acquiring information. Suppose this is the case, that when the agent does acquires information their ex-ante accuracy is q, but they only acquire information with a probability α, and the rest of the time they select option Y with probability x regardless of the realization of the value. The cost of their information outcome should then be $\alpha C_\mu(1-q,\, q) + (1-\alpha)0$, since with probability α they pay for the accuracy q, and the rest of the time their learning is costless. Following this logic one can define ex-ante accuracy cost functions for all of the relevant information outcomes (information outcomes with $\underline{s} \leq \overline{s}$).

Definition: Given a prior belief μ, the cost function C_μ is an **ex-ante cost function** for information outcomes if $\forall s \in S$ with $\underline{s} \leq \overline{s}$, $\exists\, q \in (\frac{1}{2},\, 1]$, $\exists\, x \in [0,\, 1]$, and $\exists\, \alpha \in [0,\, 1]$ such that:

$$\underline{s} = \alpha(1-q) + (1-\alpha)x,\ \overline{s} = \alpha q + (1-\alpha)x, \text{ and } C_\mu(s) = \alpha C_\mu(1-q,\, q).$$

In other words, C_μ is an ex-ante cost function for information outcomes if the cost of each information outcome is calculated as the mixture of an information outcome where the agent does not learn (so $\underline{s} = \overline{s}$), and an information outcome with symmetric ex-ante accuracies ($\underline{s} = 1 - \overline{s} > \frac{1}{2}$). In the same fashion as the previous subsection, one can wonder what behavior can be rationalized by a costly learning model if the cost function for information outcomes is restricted to be an ex-ante cost function.

Definition: Given a prior belief μ and a set of prices $\mathcal{P}$, the behavior is **rationalized by an ex-ante accuracy model** if there are values $u(\underline{\omega}) < u(\overline{\omega})$ and an ex-ante accuracy cost function for information outcomes C_μ such that, $\forall\, p \in \mathcal{P}$,

$$s(p) = (\underline{s}(p),\, \overline{s}(p)) \in \underset{(\underline{s},\, \overline{s}) \in S}{\arg\max} \Big(\underline{s}\mu(\underline{\omega})(u(\underline{\omega}) - p) + \overline{s}\mu(\overline{\omega})(u(\overline{\omega}) - p) - C_\mu(\underline{s},\, \overline{s}) \Big).$$

If the agent learns according to an ex-ante accuracy model and there is a price where the agent learns but $\Pr(\mathrm{Y}|\overline{\omega},\, p) \neq \Pr(\mathrm{X}|\underline{\omega},\, p)$, one can infer an optimal information outcome of the agent where $\underline{s} = 1 - \overline{s}$. This is shown formally by Lemma 1 in Appendix 2.

What Lemma 1 says is that if the agent learns at a pair of p and μ, and $s(p)$ is not such that $\overline{s}(p) \neq 1 - \underline{s}(p)$, then an optimal $s(p) = (1 - q,\, q)$ can be found by drawing a line from whichever of $(0,\, 0)$ and $(1,\, 1)$ is closer to $s(p)$, through $s(p)$, to the line where $\overline{s}(p) = 1 - \underline{s}(p)$ (relevant line segment is blue line in Figure 2). The point where this line hits the line where $\overline{s}(p) = 1 - \underline{s}(p)$ (relevant line segment is blue line in Figure 2), is an optimal information outcome for the agent.

If the behavior of the agent is rationalized by an ex-ante accuracy model, then at each $p \in \mathcal{P}$ where the agent learns define their **implicit learning accuracy**, denoted $q(p)$, to be the q that is optimal according to Lemma 1. This implicit learning accuracy helps us describe the behavior that characterizes ex-ante accuracy models in the following proposition.

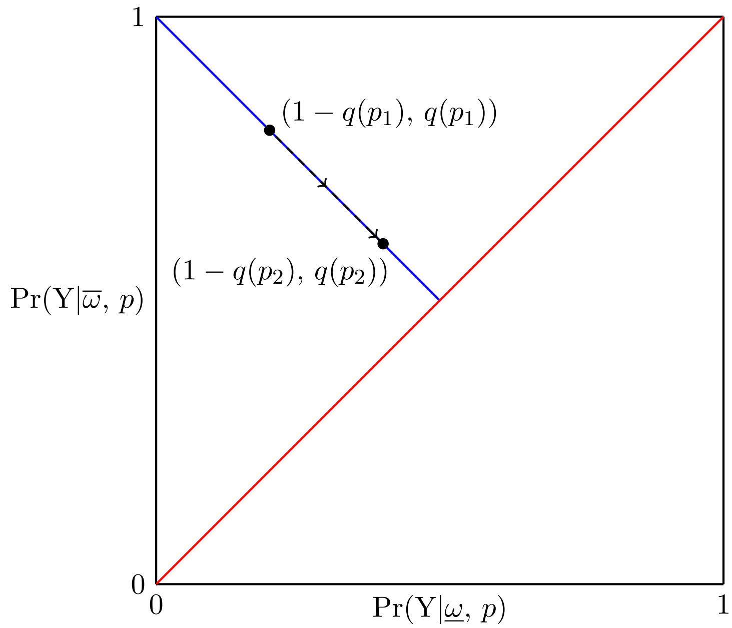
Proposition 2. Given a prior belief μ and a finite and non-empty set of prices $\mathcal{P}$, the behavior is rationalized by an ex-ante accuracy model if and only if it is rationalized by a costly learning model and the following three properties are satisfied:

(i) There is at most one price at which the agent randomizes over learning and selecting option Y without learning:

$$\text{There is at most one } p \text{ such that } \Pr(\mathrm{Y}|\overline{\omega},\, p) > \Pr(\mathrm{X}|\underline{\omega},\, p) > 0.$$

(ii) There is at most one price at which the agent randomizes over learning and selecting option X

Figure 2: Implications of ex-ante accuracy model ($\mu(\overline{\omega}) > \mu(\underline{\omega})$, $p_1 < p_2$)



without learning:

$$\text{There is at most one } p \text{ such that } \Pr(\mathrm{X}|\underline{\omega},\, p) > \Pr(\mathrm{Y}|\overline{\omega},\, p) > 0.$$

(iii) How the agent's implicit ex-ante accuracy changes is determined by the prior:

$$\text{If } \mu(\overline{\omega}) > \mu(\underline{\omega}), \text{ then } q(p) \text{ is weakly decreasing in } p \text{ wherever it is defined,}$$

$$\text{and if } \mu(\overline{\omega}) < \mu(\underline{\omega}), \text{ then } q(p) \text{ is weakly increasing in } p \text{ wherever it is defined.}$$

Proof. See Appendix 2.

Figure 2 depicts S. If I graph $s(p)$ for different $p \in \mathcal{P}$, part **(i)** requires that there be at most one p such that $s(p)$ is strictly above and to the right of the blue line in Figure 2, and part **(ii)** requires that there be at most one p such that $s(p)$ is strictly below and to the left of the blue line in Figure 2. Part **(iii)** requires that if the value $u(\overline{\omega})$ is more likely, then when price increases implicit learning accuracy decreases, as shown in Figure 2, and that if the value $u(\underline{\omega})$ is more likely, then when price increases implicit learning accuracy increases, which means it moves in the opposite direction compared to the one depicted in Figure 2.

2.3 Non-Symmetric Ex-ante Accuracy Model

In many contexts the agent may be more sophisticated than is allowed by an ex-ante accuracy model, and they may vary the relative magnitudes of their ex-ante accuracies. How could I generalize the model to accommodate for such behavior? When subjects learn I could allow them to pick their ex-ante accuracies $\underline{q} = \Pr(\mathrm{X}|\underline{\omega},\, p)$ and $\overline{q} = \Pr(\mathrm{Y}|\overline{\omega},\, p)$, independently.

If the agent wants to pick $\overline{q} = 1 - \underline{q} \in [0,\, 1]$ (so their chance of selecting option Y is the same given both realizations of the value), then they can do this without doing any learning. So, if the agent learns at a price p, I can think of them first picking $\underline{q} \in [0,\, 1]$, and then paying to pick $\overline{q} \in (1 - \underline{q},\, 1]$ according to some increasing and weakly convex function $C_{\underline{q}} : (1 - \underline{q},\, 1] \to \mathbb{R}_+$.

Definition: Given a prior belief μ, the cost function for information outcomes, C_μ, is a **non-symmetric ex-ante cost function** for information outcomes if $\forall s \in S$ with $\underline{s} \leq \overline{s}$: $C_\mu(\underline{s},\, x)$ is an increasing and weakly convex function of x for $x \in (1 - \underline{s},\, 1]$.

What behavior can be rationalized with such cost functions? As in previous subsections, the definition of 'rationalized by' stays essentially the same except for restricting the cost function for information outcomes to be a non-symmetric ex-ante cost function.

Definition: Given a prior belief μ and a set of prices $\mathcal{P}$, the behavior is **rationalized by a non-symmetric ex-ante accuracy model** if there are values $u(\underline{\omega}) < u(\overline{\omega})$ and a non-symmetric ex-ante cost function for information outcomes C_μ such that, $\forall\, p \in \mathcal{P}$,

$$s(p) = (\underline{s}(p),\, \overline{s}(p)) \in \underset{(\underline{s},\, \overline{s}) \in S}{\arg\max} \Big(\underline{s}\mu(\underline{\omega})(u(\underline{\omega}) - p) + \overline{s}\mu(\overline{\omega})(u(\overline{\omega}) - p) - C_\mu(\underline{s},\, \overline{s}) \Big).$$

Because Assumption 2 requires that all cost functions for information outcomes are convex, all cost functions for information outcomes are non-symmetric ex-ante accuracy cost functions. This means that non-symmetric ex-ante cost functions do not impose any additional structure compared to the general model of costly learning from Section 2.1, and leads to the trivial establishment of the following proposition.

Proposition 3. The behavior is rationalized by a non-symmetric ex-ante accuracy model if and only if it rationalized by a costly learning model.

Proof. Trivial.

Proposition 3 tells us that ex-ante accuracy models do not strengthen predictions for behavior if I do not impose more structure than is afforded by convexity. This is in contrast to what happens when convexity is applied to ex-post accuracy models, however, as is shown in the next section.

2.4 Ex-post Accuracy Model

Instead of modelling the agent as picking ex-ante accuracy, the agent could be modelled as picking ex-post accuracy. This means the agent selects what to learn be choosing how confident to be in their choices after they finish learning. So, they pick $\Pr(\underline{\omega}|\mathrm{X},\, p)$, the probability that option X is the better option after option X is selected, and $\Pr(\overline{\omega}|\mathrm{Y},\, p)$, the probability that option Y is the better option after option Y is selected. Ex-post and ex-ante accuracy are of course closely linked due to Bayes' Rule, but interestingly, the structure imposed on behavior by a convex cost function is substantially different when I change the object on which I am applying convexity. Pointing this dichotomy out in the context of price change is one of the theoretical contributions of this paper. The ex-post accuracy model that is studied in this subsection is similar to the Uniformly Posterior Separable model that Caplin et al. (2017) study, except I allow for the cost function to be weakly convex instead of assuming strict convexity.

To apply an ex-post model I need a function that measures how 'informed' different posteriors are. Denote this weakly convex function $c : [0,\, 1] \to \mathbb{R}$. When the agent learns they pay for the information based on the change in c it creates. Again, weak convexity of c ensures that more information (in a Blackwell (1953) sense) is weakly more costly.

Given belief μ, if the agent learns at a price p and their behavior is rationalized by a costly learning model, then $\Pr(\overline{\omega}|\mathrm{X},\, p) < \mu(\overline{\omega}) < \Pr(\overline{\omega}|\mathrm{Y},\, p)$, and I can compute the unconditional probabilities of the agent choosing the two options by solving the following equation:

$$\Pr(\mathrm{Y}|\, p)\Pr(\overline{\omega}|\mathrm{Y},\, p) + \Pr(\mathrm{X}|\, p)\Pr(\overline{\omega}|\mathrm{X},\, p)$$

$$\equiv \Pr(\mathrm{Y}|\, p)\Pr(\overline{\omega}|\mathrm{Y},\, p) + (1 - \Pr(\mathrm{Y}|\, p))\Pr(\overline{\omega}|\mathrm{X},\, p) = \mu(\overline{\omega}).$$

Then, when the agent learns at a price p in the ex-post model they pay the following cost for the information:

$$\Pr(\mathrm{Y}|\, p)\Big(c\Big(\Pr(\overline{\omega}|\mathrm{Y},\, p)\Big) - c\Big(\mu(\overline{\omega})\Big)\Big)$$

$$+\Pr(\mathrm{X}|p)\Big(c\Big(\Pr(\overline{\omega}|\mathrm{X},\, p)\Big) - c\Big(\mu(\overline{\omega})\Big)\Big),$$

which is the probability of them choosing Y times the 'increase in information' (as measured by c)

that occurred before they selected Y, plus the chance of them choosing option X times the 'increase in information' (as measured by c) that occurred before they selected X.[6]

There is a long-standing economic literature that studies dynamic information acquisition (e.g., Wald, 1945). In such models agents gradually accumulate information over multiple periods, paying for information as they learn until they decide to stop and choose one of their available options. More recent works have established equivalence results between dynamic models and static models like the ones studied in this section that model the agent as selecting ex-post accuracies (e.g., Hébert & Woodford, 2017; Morris & Strack, 2019). So, instead of conducting dynamic analysis, one can generally find a static information cost function c that is behaviorally equivalent to dynamic analysis.

The intuition for the ex-post accuracy approach that is provided by the aforementioned dynamic analysis is that the agent chooses how sure to become before they stop learning. That is, the agent chooses how sure to become of the value of option Y being high before they select option Y, and how sure to become of the value of option Y being low before they select option X. This means they pick the posterior beliefs $\Pr(\overline{\omega}|\mathrm{Y},\, p)$ and $\Pr(\underline{\omega}|\mathrm{X},\, p)$ (which dictates $\Pr(\overline{\omega}|\mathrm{X},\, p)$).

One commonly used function for measuring the 'informativeness' of a posterior is Shannon Entropy (Shannon, 1948). In models of rational inattention Shannon Entropy can be used to provide a measure of uncertainty, or the lack of information, and when it is applied the cost of an information outcome is typically measured by the expected reduction in Shannon Entropy it creates (e.g., Matějka & McKay, 2015). This is equivalent to measuring the increase in information the learning creates because the negative of Shannon Entropy can be taken as a measure of informativeness.

Definition: Given a prior belief μ, the cost function for information outcomes C_μ is an **ex-post cost function** for information outcomes if there is a weakly convex **posterior separable cost function** $c : [0,\, 1] \to \mathbb{R}$ such that $\forall s \in S$ with $\underline{s} < \overline{s}$:

$$C_\mu(\underline{s},\, \overline{s}) = \Pr(\mathrm{Y}|s)\left(c\left(\frac{\overline{s}\mu(\overline{\omega})}{\overline{s}\mu(\overline{\omega}) + \underline{s}\mu(\underline{\omega})}\right) - c(\mu(\overline{\omega}))\right)$$

$$+(1 - \Pr(\mathrm{Y}|s))\left(c\left(\frac{(1-\overline{s})\mu(\overline{\omega})}{(1-\overline{s})\mu(\overline{\omega}) + (1-\underline{s})\mu(\underline{\omega})}\right) - c(\mu(\overline{\omega}))\right),$$

$$\text{with } \Pr(\mathrm{Y}|s) = \overline{s}\mu(\overline{\omega}) + \underline{s}\mu(\underline{\omega}).$$

[6] The increase in information that occurs before choosing one option can be negative, but weak convexity of c ensures the total learning cost is never negative.

Above, $\Pr(\mathrm{Y}|s)$ is the unconditional probability the agent selecting option Y when they select information outcome $s = (\underline{s}, \overline{s})$, and $1 - \Pr(\mathrm{Y}|s)$ is the unconditional probability of the agent selecting option X when they select information outcome $s = (\underline{s}, \overline{s})$.

What behavior can be rationalized with such cost functions? As in previous subsections, the definition of 'rationalized by' stays essentially the same except for restricting the cost function for information outcomes to be an ex-post cost function.

Definition: Given a prior belief μ and a set of prices $\mathcal{P}$, the behavior is **rationalized by an ex-post accuracy model** if there are values $u(\underline{\omega}) < u(\overline{\omega})$ and an ex-post cost function for information outcomes C_μ such that, $\forall\, p \in \mathcal{P}$,

$$s(p) = (\underline{s}(p), \overline{s}(p)) \in \underset{(\underline{s}, \overline{s}) \in S}{\arg\max} \Big(\underline{s}\mu(\underline{\omega})(u(\underline{\omega}) - p) + \overline{s}\mu(\overline{\omega})(u(\overline{\omega}) - p) - C_\mu(\underline{s}, \overline{s}) \Big).$$

Proposition 4. Given a prior belief μ and a finite and non-empty set of prices $\mathcal{P}$, the behavior is rationalized by an ex-post accuracy model if and only if it is rationalized by a costly learning model and $\Pr(\overline{\omega}|\mathrm{X}, p)$ and $\Pr(\overline{\omega}|\mathrm{Y}, p)$ are both weakly increasing over the set of p where the agent learns.

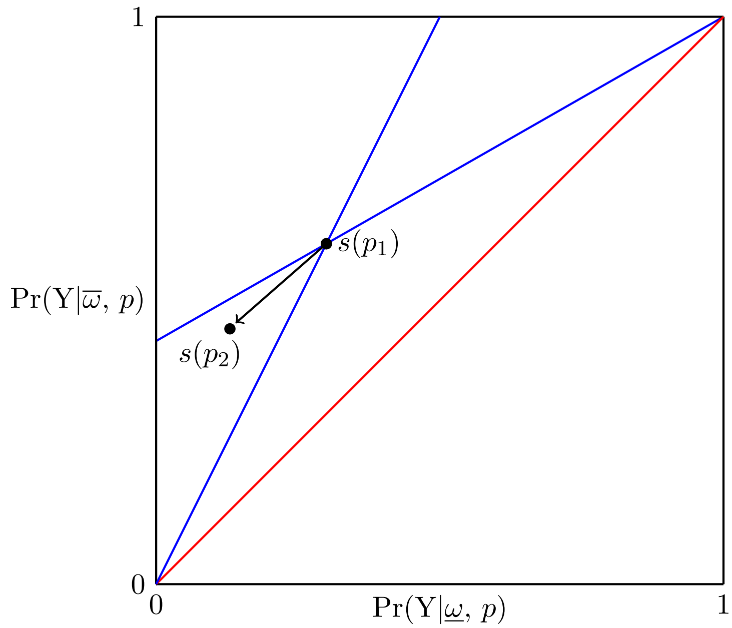
Proof. See Appendix 2.[7]

Proposition 4 says that when p increases the agent is more confident they have made the right decision when selecting option Y (more confident the value is $u(\overline{\omega})$), and less confident they made the right decision when selecting option X (less confident the value is $u(\underline{\omega})$). Proposition 4 differs from previous results because it deals with changes in ex-post accuracy, not ex-ante accuracy.

Figure 3 depicts S, and the changes in ex-ante accuracy that are equivalent to the changes in ex-post accuracy outlined in Proposition 4 (easily derived using Bayes' Rule). The blue lines are the lines through $s(p_1)$ from (0, 0) and (1, 1), and help one to visualize the proportional changes that are about to be described. When the agent is learning and price increases, $\Pr(\mathrm{Y}|\underline{\omega}, p)$ and $\Pr(\mathrm{Y}|\overline{\omega}, p)$ must both weakly *decrease*, but the proportional reduction in $\Pr(\mathrm{Y}|\underline{\omega}, p)$ must be weakly larger than the proportional reduction in $\Pr(\mathrm{Y}|\overline{\omega}, p)$, so $s(p, \mu)$ must remain weakly above the blue line from (0, 0) through $s(p)$. Further, $\Pr(\mathrm{X}|\underline{\omega}, p)$ and $\Pr(\mathrm{X}|\overline{\omega}, p)$ must both weakly *increase*, but the proportional increase in $\Pr(\mathrm{X}|\underline{\omega}, p)$ must be weakly smaller than the proportional increase in $\Pr(\mathrm{X}|\overline{\omega}, p)$, so $s(p, \mu)$ must remain weakly below the blue line from (1, 1) through $s(p)$ (assuming I

[7] Ambuehl (2017) establishes necessity of the two conditions in a context with a posterior separable function c that is strictly convex and smooth.

Figure 3: Implications of ex-post accuracy model ($p_1 < p_2$)



am not starting from $s(p, \mu) = (1,\, 1)$). In Figure 3, this means the new information outcome $s(p_2)$ must be below and to the left of the old information outcome $s(p_1)$, weakly between the blue lines.

Corollary 1. Given a prior belief μ and a finite and non-empty set of prices $\mathcal{P}$, suppose the behavior is rationalized by a costly learning model. If I graph $s(p)\, \forall\, p \in \mathcal{P}$, and then take the concave closure of these points and $(0,\, 0)$ and $(1,\, 1)$ (minimal concave function that is weakly above all points), then if $\forall\, p \in \mathcal{P}$ the information outcome $s(p)$ is on the concave closure, it is sufficient but not necessary for the behavior to be rationalized by an ex-post accuracy model.

Proof. Trivial.

If I were to graph the behavior of the agent in S for a for all prices $\mathcal{P} = \mathbb{R}$, some readers might guess that the graph is a concave function, and might wonder what this says about implicit learning costs. Corollary 1 tells us that behavior consistent with a concave function in S is consistent with a strict subset of ex-post accuracy models.

2.5 Exogenous Learning Model

Since one standard approach is to model agents as learning in an exogenous fashion, it may be informative to understand the implications of an exogenous learning model for choice behavior when price changes.

Exogenous learning does not mean I assume the agent always selects the same information outcome. One could model an agent that receives a signal with several realisations that provide the agent with information about the realized ω. In an exogenous learning model the agent does not have any control over the signal structure, which is the joint distribution between signals and values, but if price changes there may be certain realizations of the signal at which the agent chooses to switch which option they are selecting.

This paper instead models the agent as learning in an exogenous fashion by imposing that the cost of each information outcome they select at some $p \in \mathcal{P}$ has a cost of zero. If the learning of the agent is 'exogenous,' and is not changing when price changes, then one should not require changes in the cost of learning to rationalize their behavior. These two modelling strategies are equivalent in the context of this paper.

Definition: Given a prior belief μ, the behavior is **rationalized by an exogenous learning model** if there are values $u(\underline{\omega}) < u(\overline{\omega})$ and a cost function for information outcomes C_μ such that, $\forall\, p \in \mathcal{P}$,

$$s(p) = (\underline{s}(p),\, \overline{s}(p)) \in \underset{(\underline{s},\, \overline{s}) \in S}{\arg\max} \Big(\underline{s}\mu(\underline{\omega})(u(\underline{\omega}) - p) + \overline{s}\mu(\overline{\omega})(u(\overline{\omega}) - p) - C_\mu(\underline{s},\, \overline{s}) \Big).$$

$$\text{and } C_\mu(\underline{s}(p),\, \overline{s}(p)) = 0.$$

As it turns out, the behavior that can be rationalized by an exogenous learning model is a strict subset of the behavior that can be rationalized by an ex-post accuracy model. This means the ex-post accuracy model is a generalization of exogenous learning. Readers who think that an exogenous learning model generally does well predicting behavior should thus find the predictions of the ex-post model compelling.

Corollary 2. Given a prior belief μ and a finite and non-empty set of prices $\mathcal{P}$, if the behavior is rationalized by an exogenous learning model, then it is rationalized by an ex-post accuracy model.

Proof. See Appendix 2.

2.6 Aggregate Ex-post Accuracy Model

In many settings the structure imposed by an ex-post modelling approach is compelling, but datasets used for analysis do not typically focus on an individual agent. Typically datasets aggregate behavior over agents with potentially heterogeneous (accurate) prior beliefs and costs for information outcomes. Even when a dataset does feature observations from an individual, that individual might have varying private pieces of information, and a potentially varying information cost function, either due to something akin to fatigue, or variation in the choice environment that is not observable to the researcher, e.g. variation in the number of store clerks on the floor when the individual makes the decision.

If agents have different (accurate) prior beliefs and information cost functions, how does this impact how choice behavior can change when price changes? This subsection characterize an aggregate version of the ex-post accuracy model from Section 2.4, and shows that aggregating choice data from heterogeneous ex-post agents can result in behavior that cannot be rationalized with an ex-post accuracy model.

Informally, behavior is rationalized by an aggregate ex-post accuracy model if there are different types of agent, each with their own probability of occuring, prior belief, and ex-post cost function for information outcomes, such that if behavior is averaged over the different types, the observed behavior is obtained. Note, because the prior beliefs of the different types can differ, the probability of a given type being observed can differ across realizations of the value.

Definition: Given a prior belief μ, the behavior is **rationalized by an aggregate ex-post accuracy model** if there are values $u(\underline{\omega}) < u(\overline{\omega})$, $T \in \mathbb{N}$ types of agent each of which has probability of occurring $\pi_t > 0$, a belief about the probability of $\overline{\omega}$ occurring $\mu_t(\overline{\omega}) \in [0, 1]$, and behavior s_t, such that each type's behavior is rationalized by an ex-post accuracy model with values $u(\underline{\omega}) < u(\overline{\omega})$, and:

(i) The probabilities of the different types sum to one, and their mean belief is the distribution over values observed by the researcher:

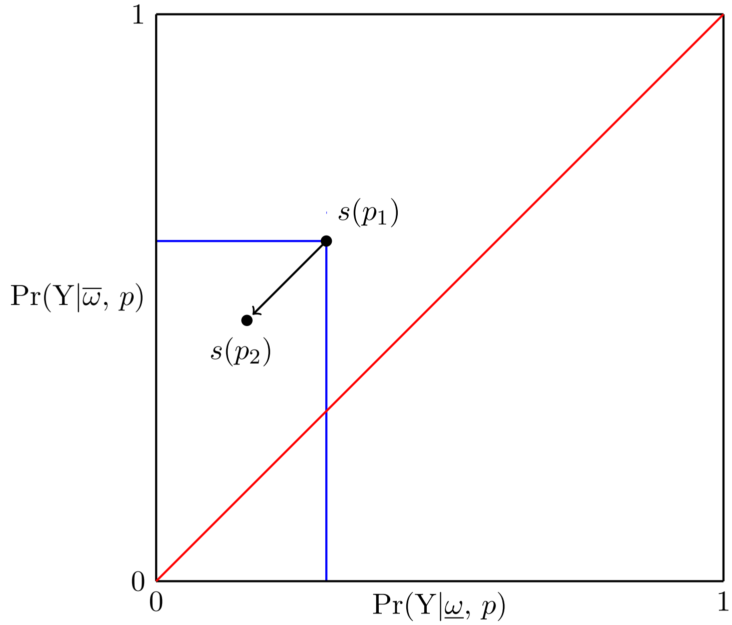
$$\sum_{t=1}^{T} \pi_t = 1, \;\; \sum_{t=1}^{T} \mu_t(\overline{\omega})\pi_t = \mu(\overline{\omega}).$$

(ii) Given any pair of price p and value $u(\omega)$, the mean behavior is the behavior observed by the researcher:

$$\text{and } \forall\, p \in \mathcal{P},\, \forall\, \omega \in \Omega : \sum_{t=1}^{T} \Pr_t(\mathrm{Y}|\omega,\, p)\frac{\mu_t(\omega)\pi_t}{\mu(\omega)} = \Pr(\mathrm{Y}|\omega,\, p).$$

Proposition 5. Given a prior belief μ and a finite and non-empty set of prices $\mathcal{P}$, the behavior is rationalized by an aggregate ex-post accuracy model if and only if it is rationalized by a costly learning model and $\Pr(\mathrm{Y}|\underline{\omega},\, p)$ and $\Pr(\mathrm{Y}|\overline{\omega},\, p)$ are both weakly decreasing in p.

Figure 4: Implications of the aggregate ex-post accuracy model ($p_1 < p_2$)



Proof. See Appendix 2.

Proposition 5 says that the behavior can always be rationalized with heterogenous agents that all behave in line with ex-post accuracy models if and only if when price increases the probability of selecting option Y decreases given both realizations of the value. Figure 4 depicts S, and the changes in ex-ante accuracy described in Proposition 5. Given a value $u(\omega)$, Proposition 5 requires that on average agents are weakly less likely to select option Y when price increases, which means $\Pr(\mathrm{Y}|\underline{\omega},\, p)$ and $\Pr(\mathrm{Y}|\overline{\omega},\, p)$ are weakly decreasing in p. In Figure 4, this means that if $p_2 > p_1$ the new information outcome $s(p_2)$ is below and to the left of the old information outcome $s(p_1)$, weakly between the two blue lines.

Using Bayes' Rule, it is easy to show that such behavior can violate the predictions of Proposition 4, as is stated in the following corollary. Further, one can visually see the difference in predicted behavior when ex-post agents are aggregated by comparing Figure 3 and Figure 4. As a result, aggregating over agents that behave in line with an ex-post accuracy model can produce behavior that cannot be rationalized by an ex-post accuracy model. Heterogeneity of agents is thus significant for understand how choices can change when price changes.

Corollary 3. Given a prior belief μ, suppose the behavior is rationalized by an aggregate ex-post accuracy model. It is possible that $\Pr(\overline{\omega}|\mathrm{Y},\, p)$ or $\Pr(\overline{\omega}|\mathrm{X},\, p)$ decrease when p increases.

Proof. Follows from Proposition 5 and Bayes' Rule.

What is the intuition behind this result? Suppose some agents have a high cost of learning and a high apriori chance that option Y is beneficial for them, while others have a low cost of learning and low chance that option Y is beneficial for them. If p is low, the agents with the high learning cost may select option Y without learning, and may have a fairly high chance of regret ex-post, while the individuals with the low cost for learning might have incentive to learn, and perhaps only choose option Y if they are quite sure it is beneficial for them to do so. If p is increases, then for the individuals that have a low chance of option Y being beneficial and a low learning cost, the low chance of option Y being beneficial may dominate the low cost of learning, and they may choose option X without doing any learning. The individuals with the high learning cost, however, may still select option Y without learning, and may still have a fairly high chance of regret ex-post. The outcome could then be that after an agent selects option Y there is a higher chance of ex-post regret when the price is higher, which means $\Pr(\overline{\omega}|\mathrm{Y}, p)$ decreases when p increases.

2.7 Aggregate Ex-ante Accuracy Model

What about an aggregate ex-ante accuracy model? As it turns out, compared to an aggregate ex-post accuracy model, an aggregate ex-ante accuracy model imposes strictly less structure on behavior. Proposition 2 tells us that, given a particular realization of the value $u(\omega)$, the probability of selecting option Y can increase when price increases in an ex-ante accuracy model, so I know that the necessary conditions from an aggregate ex-post model are not necessary for rationalization of the behavior in an ex-ante setting. It is not immediately evident that the sufficient conditions from an aggregate ex-post model are sufficient for rationalizing behavior with an aggregate ex-ante model. In the Proof of Proposition 5, however, I only used types of agents that do no learning, or perfectly learn the state of the world (choose the information outcome $s = (0, 1)$). This means that the behavior of each type of agent can be rationalized with an ex-ante accuracy model, and sufficiency follows easily.

Definition: Given a prior belief μ, the behavior is **rationalized by an aggregate ex-ante accuracy model** if there are values $u(\underline{\omega}) < u(\overline{\omega})$, $T \in \mathbb{N}$ types of agent each of which has probability of occurring $\pi_t > 0$, a belief about the probability of $\overline{\omega}$ occurring $\mu_t(\overline{\omega}) \in [0, 1]$, and behavior s_t, such that each type's behavior is rationalized by an ex-ante accuracy model with values $u(\underline{\omega}) < u(\overline{\omega})$, and:

(i) The probabilities of the different types sum to one, and their mean belief is the distribution over values observed by the researcher:

$$\sum_{t=1}^{T} \pi_t = 1, \; \sum_{t=1}^{T} \mu_t(\overline{\omega})\pi_t = \mu(\overline{\omega}).$$

(ii) Given any pair of price p and value $u(\omega)$, the mean behavior is the behavior observed by the researcher:

$$\text{and } \forall\, p \in \mathcal{P},\, \forall\, \omega \in \Omega : \sum_{t=1}^{T} \Pr_t(\mathrm{Y}|\omega,\, p)\frac{\mu_t(\omega)\pi_t}{\mu(\omega)} = \Pr(\mathrm{Y}|\omega,\, p).$$

Proposition 6. Given a prior belief μ and a finite and non-empty set of prices $\mathcal{P}$, if the behavior is rationalized by an aggregate ex-post accuracy model then it is rationalized by an aggregate ex-ante accuracy model.

Proof. See Appendix 2.

2.8 Theory Tailored to the Experiment

So far the theory results have not discussed changes in belief, only changes in price. In many contexts the beliefs of decision makers, and how they change, are not observed by the researcher so the theory results that have been introduced are pertinent. In the experiment, in contrast, I induce the belief of the subject in each decision problem so I can conduct comparative statics on belief.

In the experiment I can further observe if a subject chooses to 'get more information' in each decision problem, as is explained in the next section. This enrichment of the data is helpful for several reasons. In particular, it allows me to reject the aggregate ex-ante model even though I cannot reject the aggregate ex-post model, which is not possible with a 'standard' dataset. With a 'standard' dataset where one cannot observe if the subject chooses to 'get more information' this is not possible because, as Proposition 6 indicates, all behavior that can be rationalized with an aggregate ex-post model can be rationalized with an aggregate ex-ante model.

This paper has more theory results to introduce, but they are specifically tailored to the context of the experiment as they require the observation of preferences, changes in belief, or the decision to 'get more information.' In order to streamline the exposition, I present them after the experiment has been explained.

3 Experiment

A typical experiment on models of rational inattention estimates the probabilities of different decisions being made. Probabilities are hard to estimate because it takes a large number of observations to ensure that the observed frequency of an event occurring is sufficiently close to the true probability. This means that for statistical power, analysis typically requires aggregating choices from groups of individuals.

Even if each subject's behavior is completely in line with the ex-ante model, for instance, if choices are aggregated from multiple individuals the resultant behavior may look nothing like the ex-ante model. The ex-ante model predicts that the agent should either not learn, or should choose an information outcome that has very specific properties at all but at most two prices (see Proposition 2). The set of information outcomes that the ex-ante model predicts an agent should generically choose if they learn is not a convex set, however, so aggregation can result in behavior that does not look like the ex-ante model, even if each individual behaves in line with the ex-ante model.

In this paper's experiment information about the probability of agents selecting different options is observed, but whether or not an agent chooses to learn in each decision problem is also observed. This information is quite useful for identifying a subject's learning strategy. The choice to acquire more information results in a binary outcome, and using it for analysis does not require the estimation of a continuous variable like probability. Further, one can specifically focus on decision problems in which subjects do choose to acquire information, and estimate their accuracy in such decision problems. This produces a clean identification of what information subjects are acquiring on average when they do choose to learn, because one does not need to average choice mistakes over decision problems where subjects are and are not learning.

As was shown in Section 2, ex-ante and ex-post modelling strategies for accuracy lead to starkly different predictions for how choice accuracy changes when price changes. The experiment, which was completed by 243 undergraduate students from the University of Toronto, was designed to let me test the models against each other. In the experiment each subject faced 92 **decision problems** in which they chose between a safe option X and an uncertain option Y, and, if they desired, they could also choose to 'get more information' before selecting an option, so as to try to deduce the payoff from the uncertain option Y.

In each of these decision problems the payoff from option X is always a $p \in \{0.25, 0.5\}$, which is clearly displayed to the subject. In the first 40 and last 40 decision problems $p = 0.25$, and in the middle 12 decision problems (decision problems 41 through 52) $p = 0.5$. The payoff of option Y is a $u(\omega) \in \{0, 1\}$, which is not clearly displayed. These payoffs are equivalent to the payoffs studied in the theory part of the paper, since if p is subtracted from both payoffs in each decision problem one obtains the payoffs studied in the theoretical results. These payoffs are used for the experiment so that subjects do not need to worry about their choice in a decision problem resulting in negative payoffs.

The payoffs of the options are measured in terms of percentage points. Subjects pick options to try to gradually increase their chance of winning a monetary prize at the end of the experiment. For example, if a subject always selects option Y, and in 50 decision rounds it has a payoff of 1, and the rest of the time it has a payoff of 0, then they have a 50% chance of winning the prize when the experiment ends. The prize the subject can win in the draw is either \$20, \$25, or \$30, depending on the subject's treatment group.[8]

In this experiment subjects are 'paid' with percentage points for the options they select in a decision problem so that within subject analysis can be conducted in an incentive compatible way. An experiment is said to be incentive compatible if the subject would have behaved in the same way in each decision problem if they had faced the decision problem on its own, instead of facing it in the context of the sequence of decision problems in the experiment (Azrieli, Chambers, & Healy, 2018). The standard strategy for achieving incentive compatibility is to randomly select a decision problem and pay the subject based on the option they selected in that decision problem (Allais, 1953). In the setting studied by this paper this strategy does not achieve incentive compatibility because the agent needs to consider their cost of learning, and they cannot defer their learning until they know which decision problem is selected. See Appendix 1 for a more detailed discussion of incentive compatibility.

In each decision problem the subject is presented with a big dot that is either red or green. The big dot induces the belief of the subject about the payoff from option Y. If the big dot is red there is a $\frac{1}{4}$ chance that option Y is the better option ($\mu(1) = \frac{1}{4}$), and if the big dot is green there is a $\frac{3}{4}$ chance that option Y is the better option ($\mu(1) = \frac{3}{4}$). In each decision problem there is a $\frac{3}{4}$ chance that the big dot is red, and a $\frac{1}{4}$ chance that the big dot is green.

Figure 5 displays an example of a decision problem, and Figure 6 displays the same decision

[8]The variation in the prize size was a design choice that after the fact seemed irrelevant relative to the apparent variation in how costly it is for agents to learn. See Section 4.5.

Figure 5: Option X, Option Y, or Get more information:

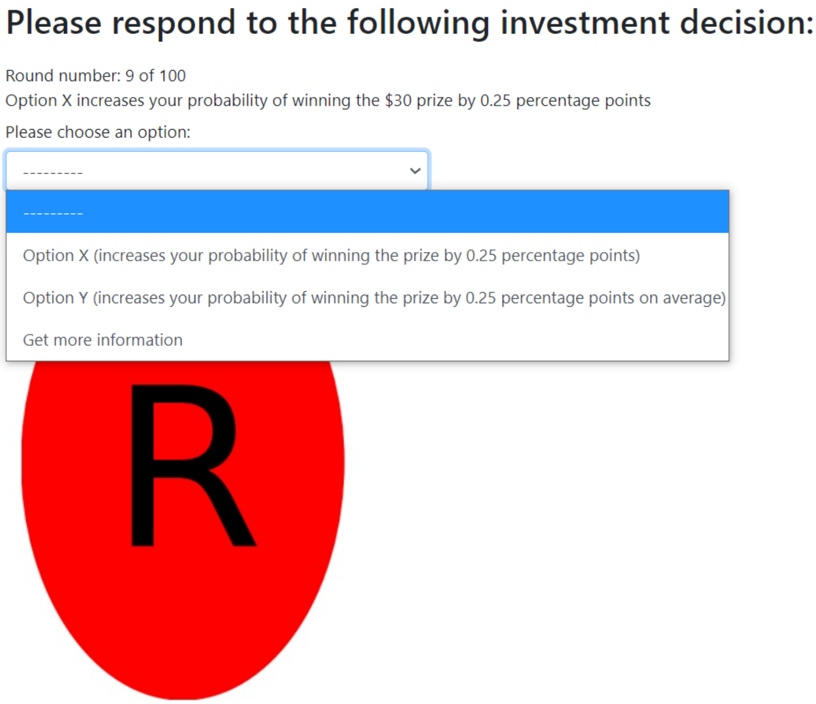
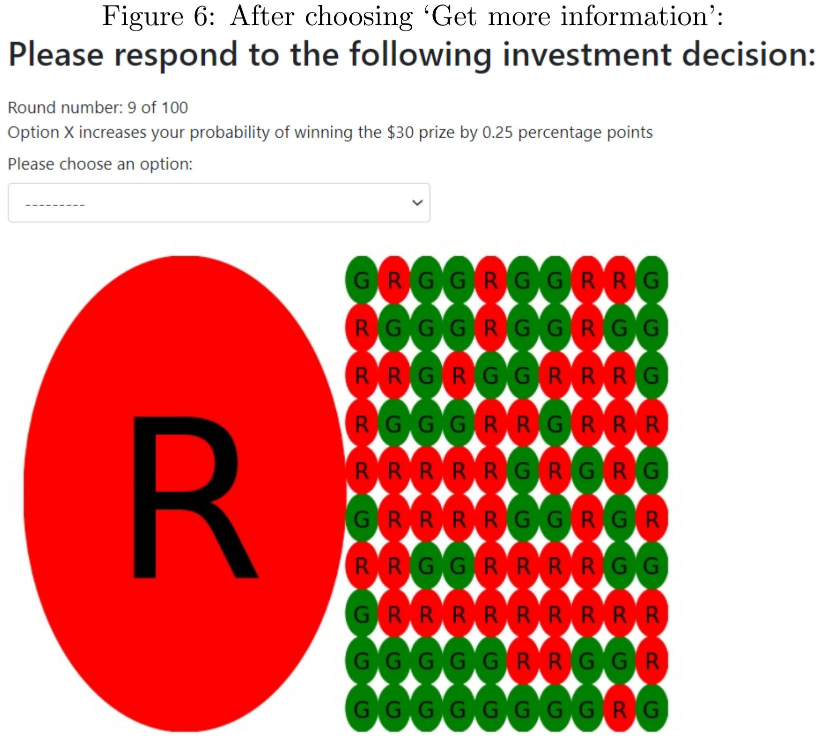


Figure 6: After choosing 'Get more information':



problem after the subject has chosen to 'get more information.' If a subject chooses to 'get more information,' 100 small dots appear, each of which is either red or green. There are always 49 of one color of small dot, and 51 of the other color. If 51 of the small dots are red, then the payoff from option Y is 0, and if 51 of the small dots are green, then the payoff from option Y is 1.

Subjects participated on-line due to COVID-19.[9] In addition to the decision problems discussed above, each subject was trained, completed a quiz, and went through a test for the reduction of compound lotteries (first eight 'rounds' before the 92 rounds of decision problems). For a more detailed description of the experiment, see Appendix 1.

In the context of the experiment the ex-ante accuracy model is quite compelling. Suppose a subject learns by counting the small dots of one color, either red or green. If their count is unbiased then the probability of under-counting when there are 51 small dots of the color, and the probability of over counting when there are 49 small dots of the color, may be the same. At the very least this is a reasonable hypothesis to test. Further, the probability of counting 50 small dots of the color might be the same when there are in fact either 49 or 51 small dots of the color, in which case it is an uninformative signal, and they would presumably have incentive to count again. If this is true, and the subject counts accurately enough that they stop counting and pick an option when they count anything other than 50 small dots of the color, and counts again in the same way when they do count 50 small dots of the color, then the subject's two ex-ante accuracies would be the same each time they learn.

3.1 The Decision to 'get more information'

So far there has not been a discussion about what the theory says about the choice to 'get more information.' In many contexts the researcher cannot observe if an agent selects an option based on their prior belief or tries to acquire more information before deciding, and the results from Section 2 that focus on behavior are more appropriate in such settings. In the experiment the outcome of the decision to 'get more information' is observed, however, and this additional data can be quite informative. A natural place to start analysis is predicting the situations in which, according to the theory, a subject should be more likely to choose to 'get more information.'

Proposition 7. Given a prior belief μ, and values $u(\underline{\omega}) < u(\overline{\omega})$, suppose the behavior is rationalized by a costly learning model with cost function for information outcomes C_μ, and that there are two

[9] The experiment was programmed using oTree (Chen, Schonger, & Wickens, 2016).

prices $p_1,\, p_2 \in \mathcal{P}$.

(i) If the prices are below the expected value of ω, i.e. $p_1 < p_2 \leq \mu(\underline{\omega})u(\underline{\omega}) + \mu(\overline{\omega})u(\overline{\omega})$, then the subject learns at p_2 if they learn at p_1:

$$\Pr(\mathrm{Y}|\underline{\omega},\, p_1) \neq \Pr(\mathrm{Y}|\overline{\omega},\, p_1) \Rightarrow \Pr(\mathrm{Y}|\underline{\omega},\, p_2) \neq \Pr(\mathrm{Y}|\overline{\omega},\, p_2).$$

(ii) If the prices are above the expected value of ω, i.e. $\mu(\underline{\omega})u(\underline{\omega}) + \mu(\overline{\omega})u(\overline{\omega}) \leq p_1 < p_2$, then the subject learns at p_1 if they learn at p_2 :

$$\Pr(\mathrm{Y}|\underline{\omega},\, p_2) \neq \Pr(\mathrm{Y}|\overline{\omega},\, p_2) \Rightarrow \Pr(\mathrm{Y}|\underline{\omega},\, p_1) \neq \Pr(\mathrm{Y}|\overline{\omega},\, p_1).$$

Proof. Suppose the agent learns at p_1. If $p_1 < p_2 \leq \mu(\underline{\omega})u(\underline{\omega}) + \mu(\overline{\omega})u(\overline{\omega})$, then when p increases, the payoff from $s(p_1)$ strictly decreases, but not as quickly as the payoff from not learning and selecting option Y, so the agent must learn at p_2.

Suppose the agent learns at p_2. If $\mu(\underline{\omega})u(\underline{\omega}) + \mu(\overline{\omega})u(\overline{\omega}) \leq p_1 < p_2$, then when p decreases the payoff from $s(p_2)$ increases while the payoff from not learning and selecting X stays the same, so the agent must learn at p_1. ■

What are the implications of Proposition 7 for the experiment? If the cost function for information C_μ does not change when p does, then if the big dot is red and a subject chose to 'get more information' when $p = 0.5$, then they must always choose to 'get more information' when the big dot is red and $p = 0.25$, while if the big dot is green and a subject chose to 'get more information' when $p = 0.25$, then they must always choose to 'get more information' when the big dot is green and $p = 0.5$.

When looking at aggregate data, Proposition 7 implies that if the distribution of the cost function for information C_μ does not change when p does, then if the big dot is red subjects should be weakly more likely to choose to 'get more information' when $p = 0.25$, while if the big dot is green subjects should be weakly more likely to 'get more information' when $p = 0.5$.

Even if the cost function for information changes when the belief does, it may be compelling to impose symmetry on how it changes, since the prior beliefs in this paper's experiment are symmetrically located on either side of $\frac{1}{2}$.

To gain intuition for why symmetry of cost functions for information outcomes is compelling when belief changes, suppose $p = 0.5$. When the big dot is red if the subject does not choose to 'get more information' their default decision, the option that is optimal if they do not acquire any information, is option X, which has a 75% chance of being more valuable than option Y by 0.5, and a 25% chance of being less valuable than option Y by 0.5. When the big dot is green if the subject does not choose to 'get more information' their default decision is option Y, which has a 75% chance of being more valuable than option X by 0.5, and a 25% chance of being less valuable than option X by 0.5. So, there is a lot of symmetry when the subject does not 'get more information.'

When the agent chooses to 'get more information,' there should also be a great deal of symmetry. Again, suppose $p = 0.5$. The agent's learning results in a chance that they select the default option (the default option is determined by the color of the big dot) when it is the better option and a chance that they do not select the default option when it is not the better option. When one frames the learning of the agent in this way, it does not seem like the cost or benefit of a given information outcome should depend on the color of the big dot.

So, if $\underline{\mu}$ is the prior belief such that $\underline{\mu}(1) = \frac{1}{4}$, and $\overline{\mu}$ is the prior belief such that $\overline{\mu}(1) = \frac{3}{4}$, then one might expect that $\forall s \in S$ with $\underline{s} \leq \overline{s}$: $C_{\underline{\mu}}(\underline{s}, \overline{s}) = C_{\overline{\mu}}(1-\overline{s}, 1-\underline{s})$. If this is the case, then if $p = 0.5$ the incentive for subjects to choose to 'get more information' should be the same when the big dot is red as when the big dot is green.

Proposition 8. Given two prior beliefs $\underline{\mu}$ and $\overline{\mu}$ that are symmetrically located on either side of $\frac{1}{2}$, i.e. $\underline{\mu}(\overline{\omega}) = \overline{\mu}(\underline{\omega}) < \frac{1}{2} < \underline{\mu}(\underline{\omega}) = \overline{\mu}(\overline{\omega})$, values $u(\underline{\omega}) < u(\overline{\omega})$, and a price p that is the average of the mean values of $\underline{\mu}$ and $\overline{\mu}$,

$$\text{i.e. } \frac{\underline{\mu}(\underline{\omega})u(\underline{\omega}) + \underline{\mu}(\overline{\omega})u(\overline{\omega}) + \overline{\mu}(\underline{\omega})u(\underline{\omega}) + \overline{\mu}(\overline{\omega})u(\overline{\omega})}{2} = p,$$

if the cost functions for information outcomes are symmetric, i.e. $\forall s \in S$ with $\underline{s} \leq \overline{s}$, $C_{\underline{\mu}}(\underline{s}, \overline{s}) = C_{\overline{\mu}}(1-\overline{s}, 1-\underline{s})$, then learning at one belief is optimal if and only if it is optimal at the other belief:

$$(x,\, y) \in \underset{(\underline{s},\, \overline{s}) \in S}{\arg\max} \Big(\underline{s}\underline{\mu}(\underline{\omega})(u(\underline{\omega}) - p) + \overline{s}\underline{\mu}(\overline{\omega})(u(\overline{\omega}) - p) - C_{\underline{\mu}}(\underline{s},\, \overline{s}) \Big)$$

$$\text{iff}$$

$$(1-y,\, 1-x) \in \underset{(\underline{s},\, \overline{s}) \in S}{\arg\max} \Big(\underline{s}\overline{\mu}(\underline{\omega})(u(\underline{\omega}) - p) + \overline{s}\overline{\mu}(\overline{\omega})(u(\overline{\omega}) - p) - C_{\overline{\mu}}(1-\overline{s},\, 1-\underline{s}) \Big).$$

Proof. See Appendix 2.

What are the implications of Proposition 7 and Proposition 8 for the experiment? If the distribution of the cost function for information C_μ does not change when p does, and changes in a symmetric way or does not change when μ does, then subjects should be the most likely to learn when the big dot is red and $p = 0.25$, and the least likely to learn when the big dot is green and $p = 0.25$.

4 Experimental Results

As is shown in Section 2, modelling an agent as choosing ex-post or ex-ante accuracies leads to contrasting implications for how choice behavior changes when price changes, and in either model aggregation over heterogeneous agents could have significantly different consequences for behavior. The experiment is designed to provide me with an opportunity to evaluate the models against each other. Because it is observed if subjects chose to 'get more information' in each decision problem, it is easier to determine which model best describes subjects. Further, with this enrichment the data may reject the aggregate version of the ex-ante model without rejecting the aggregate version of the ex-post model, which Proposition 6 tells us is not possible with standard data.

4.1 Changes in Price: Data

When p changes the predictions of the different models can be evaluated for a fixed big dot, or when data is aggregated over decision problems with red and green big dots. In all three of these scenarios, when behavior is aggregated over subjects, and whether or not they chose to 'get more information' is ignored, a statistically significant rejection of the ex-ante model described in Proposition 2 is obtained,[10] but behavior is completely in line with the predictions of the ex-post model described in Proposition 4. This is all demonstrated by Table 1, Table 2, Table 3, and the associated graphs that can be viewed by clicking the hyper-links in the tables (or be found in the last three pages of the paper). The associated graphs evaluate the behavior in the same fashion as how graphs are used to explain the propositions in Section 2.

The ex-post model is successful even though the experiment seems to be quite conducive to analysis with the ex-ante model. To gain insight into why the ex-ante model performs poorly in the comparative static with respect to price, it is important to recall that the ex-ante model places a lot of restrictions on the region that information outcomes can be observed, not just how they

[10] A two-sided Fisher Exact test was used to test whether or not $\underline{s} = 1 - \overline{s}$ for each data point, and each test rejected the hypothesis significantly with a p-value of 0.01. In each situation, big dot red, big dot green, or both red and green big dots, both observed information outcomes were on the same side of the blue line in Figure 2.

Table 1

Aggregate Behavior: Big Dot Red		Proportional Change
$\Pr(\mathrm{Y}\mid\underline{\omega},\, p = 0.25) = 0.182$	$\Pr(\mathrm{Y}\mid\underline{\omega},\, p = 0.5) = 0.098$	-46.2%
$\Pr(\mathrm{Y}\mid\overline{\omega},\, p = 0.25) = 0.513$	$\Pr(\mathrm{Y}\mid\overline{\omega},\, p = 0.5) = 0.391$	-23.8%
$\Pr(\mathrm{X}\mid\underline{\omega},\, p = 0.25) = 0.818$	$\Pr(\mathrm{X}\mid\underline{\omega},\, p = 0.5) = 0.902$	+10.3%
$\Pr(\mathrm{X}\mid\overline{\omega},\, p = 0.25) = 0.487$	$\Pr(\mathrm{X}\mid\overline{\omega},\, p = 0.5) = 0.609$	+25.1%
Contradicts Prop. 2 (ex-ante model)		Consistent with Prop. 4 (ex-post model)

Table 2

Aggregate Behavior: Big Dot Green		Proportional Change
$\Pr(\mathrm{Y}\mid\underline{\omega},\, p = 0.25) = 0.645$	$\Pr(\mathrm{Y}\mid\underline{\omega},\, p = 0.5) = 0.511$	-20.8%
$\Pr(\mathrm{Y}\mid\overline{\omega},\, p = 0.25) = 0.943$	$\Pr(\mathrm{Y}\mid\overline{\omega},\, p = 0.5) = 0.893$	-5.3%
$\Pr(\mathrm{X}\mid\underline{\omega},\, p = 0.25) = 0.355$	$\Pr(\mathrm{X}\mid\underline{\omega},\, p = 0.5) = 0.489$	+37.7%
$\Pr(\mathrm{X}\mid\overline{\omega},\, p = 0.25) = 0.057$	$\Pr(\mathrm{X}\mid\overline{\omega},\, p = 0.5) = 0.107$	+87.7%
Contradicts Prop. 2 (ex-ante model)		Consistent with Prop. 4 (ex-post model)

change. Generically, the ex-ante model predicts information outcomes are on the blue line from Figure 2, or are (0, 0) or (1, 1). If data aggregates behavior from *heterogeneous* agents, even if their information outcomes are individually in the prescribed region, the average behavior is typically not, as the prescribed region is not convex. As can be seen in Figure 7, at the aggregate level, subjects consistently randomize over learning and not learning for different pairs of p and μ. This is not consistent with an ex-ante accuracy model (e.g. see Section 4.3).

Is it surprising that the data does not reject the ex-post model when behavior is aggregated over individuals with different prior beliefs? Perhaps. If subjects choose to 'get more information'

Table 3

Aggregate Behavior: Red and Green Big Dots		Proportional Change
$\Pr(\mathrm{Y}\mid\underline{\omega},\, p = 0.25) = 0.224$	$\Pr(\mathrm{Y}\mid\underline{\omega},\, p = 0.5) = 0.135$	-39.7%
$\Pr(\mathrm{Y}\mid\overline{\omega},\, p = 0.25) = 0.741$	$\Pr(\mathrm{Y}\mid\overline{\omega},\, p = 0.5) = 0.566$	-23.6%
$\Pr(\mathrm{X}\mid\underline{\omega},\, p = 0.25) = 0.776$	$\Pr(\mathrm{X}\mid\underline{\omega},\, p = 0.5) = 0.865$	+11.5%
$\Pr(\mathrm{X}\mid\overline{\omega},\, p = 0.25) = 0.259$	$\Pr(\mathrm{X}\mid\overline{\omega},\, p = 0.5) = 0.434$	+67.6%
Contradicts Prop. 2 (ex-ante model)		Consistent with Prop. 4 (ex-post model)

when their incentive to do so is maximized – when the big dot is red and $p = 0.25$, and do not otherwise, then given any strictly positive probabilities of big red dots and big green dots (independent of p), as long as the subjects learn accurately enough when they 'get more information,' behavior should be observed that cannot be rationalized by an ex-post accuracy model.

The intuition for this is that if the probability of option Y having a high value is high and price is low, then a moderate increase in price does not change the behavior of the agent (since the agent does not chooses to 'get more information' and instead chooses option Y without learning), whereas if the probability of option Y having a high value is low and price is low, then a moderate increase in p would result in $\Pr(\mathrm{Y}|\overline{\omega}, p)$ dropping (since the subject no longer chooses to 'get more information' and instead chooses option X without learning) while $\Pr(\mathrm{Y}|\underline{\omega}, p)$ stays essentially the same. On the aggregate level, if the agent is learning arbitrarily accurately enough when they do learn, the change in the aggregate $s(p)$ would be arbitrarily close to a vertical drop, which is inconsistent with the ex-post accuracy model described in Proposition 4.

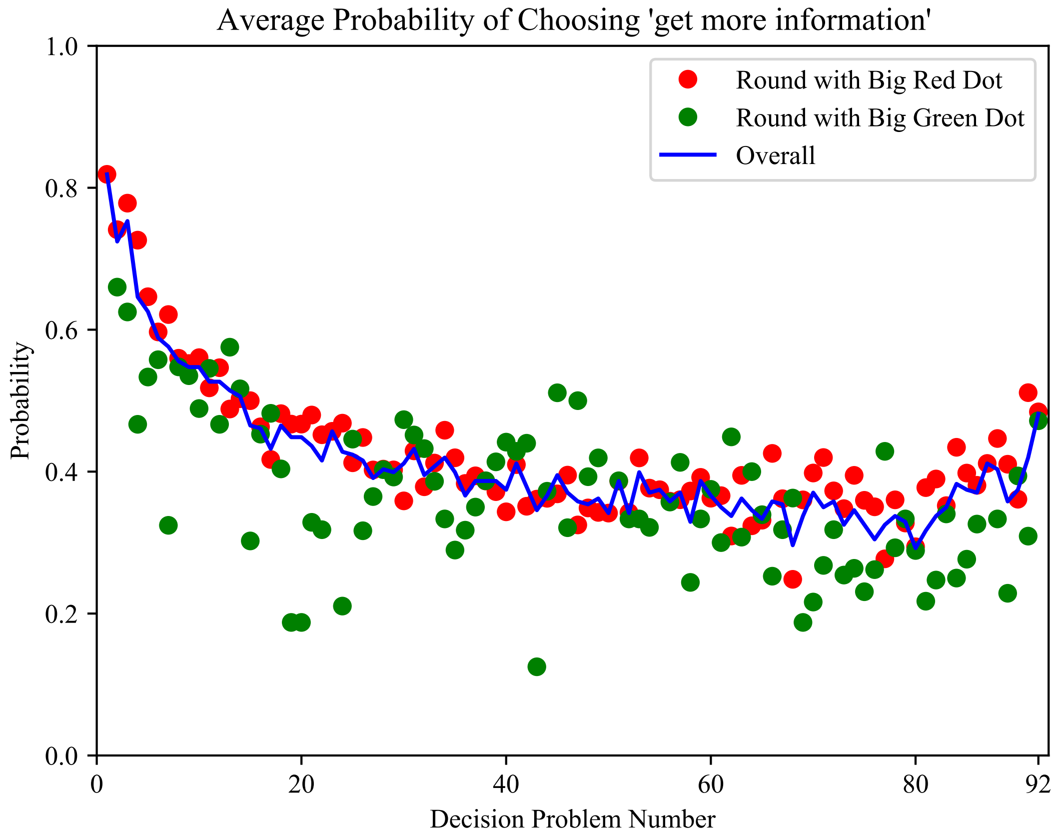
But this did not happen in the experiment. Again, an explanation can be found in the heterogeneity of subjects. Given each pair of p and μ, there are significant chances of not learning and learning, as can be seen in Figure 7, and is discussed in the next sub-section. This is inconsistent with a representative agent version of the ex-post accuracy model, but results in behavior that is.

4.2 Decision to 'get more information:' Data

This subsection tests the implications of Section 3.1. Remember, in each decision problem the big dot color is red with a $\frac{3}{4}$ chance, and is green with a $\frac{1}{4}$ chance. In the first 40 and last 40 decision problems a subject faces $p = 0.25$. Thus, comparing the probabilities of choosing to 'get more information' when $p = 0.25$ and the big dot is either red or green is straightforward because the observed distribution of decision problems in which these pairings are occurring are roughly the same, and I do not need to worry much about issues such as 'fatigue.' Comparing across prices is more challenging because the rounds in which $p = 0.5$ differ from those in which $p = 0.25$, and fatigue is more of a concern. The middle 12 rounds have $p = 0.5$, but it is not necessarily the case that 'average' fatigue is the same in the decision problems where $p = 0.25$ and $p = 0.5$, even though their mean decision problem numbers are the same.

In total, subjects chose to 'get more information' in 9,233 of 22,356 decision problems. The average chance of a subject choosing to 'get more information' for a pair of p and μ is depicted in Table 4. If $p = 0.25$, there is a 43.8% chance that a subject chose to 'get more information' if the

Figure 7:



big dot is red, and a 36.5% chance that a subject chose to 'get more information' if the big dot is green. Using a two-sided Fisher Exact test shows that these chances are statistically significantly different at the 1% level, which is in line with the implications of Proposition 7 and Proposition 8 for aggregate data if the cost function changes in a symmetric way when belief changes.

If $p = 0.5$ there is a 36.8% chance that a subject chose to 'get more information.' This is statistically significantly less than the chance if $p = 0.25$ and the big dot is red,[11] but is not statistically significantly more than the chance if $p = 0.25$ and the big dot is green. All of this is in line with the implications of Proposition 7 and Proposition 8 for aggregate data. The lack of statistical significance in the latter case could be due to fatigue, or it could be due to the smaller number of observations of decision problems with a big green dot and $p = 0.25$, or it could be due to violation of the implicit assumption that subjects reduce compound lotteries, which means subjects may not always choose the option that produces the higher average increase in the chance

[11] Two-sided Fisher Exact test rejects at the 1% level.

Table 4: Probability of Choosing 'get more information'

	Big Dot Red	Big Dot Green
$p = 0.25$	43.8%	36.5%
$p = 0.50$	36.0%	39.6%

of winning the prize, even if they do not 'get more information', and subjects are perhaps more inclined to 'get more information' if $p = 0.25$ and the big dot is green compared to if $p = 0.5$ and the big dot is red. There is a more detailed discussion of the reduction of compound lotteries in Appendix 1.

Because there is positive chance of a subject choosing to 'get more information' for each pair of p and μ, Proposition 7 implies the aggregate behavior is inconsistent with any single cost function for information outcomes. Further, Proposition 7 and Proposition 8 imply the aggregate behavior is inconsistent with two cost functions for information outcomes that are symmetric (see Section 3.1). This is the first indication that heterogeneity is significant for understanding aggregate behavior, more follow.

4.3 Changes in Belief: Data

What happens if belief changes? There is even more dichotomy if the predictions of the ex-ante and ex-post models are studied in a setting with a change in belief. To impose structure on how behavior changes if belief changes, it is natural to fix a cost function for information, but fixing either C_μ or c when belief changes generically results in changing the other, so the models result in contrasting predictions.

If $p = 0.25$ and an ex-ante model is considered with a C_μ that does not change when μ does, then the behavior in decision problems with a big green dot outlined in Table 2 indicates that the agent is indifferent between learning and not learning selecting option Y with probability 1 since $\underline{s}(p) > 1 - \overline{s}(p)$ (See Lemma 1 and Proposition 2). This means the benefit from learning and choosing an optimal ex-ante accuracy $q \in (\frac{1}{2}, 1]$ compared to just choosing option Y is zero:

$$0.75(1-q)(0.25-1) + 0.25q(0.25-0) - C_\mu(1-q,\, q) = 0.$$

However, the behavior in decision problems with a big red dot outlined in Table 1 also indicates that the agent is indifferent between learning and not learning selecting option X with probability 1 since $\underline{s}(p) < 1 - \overline{s}(p)$ (See Lemma 1 and Proposition 2). But, given the above equation, the benefit from learning and choosing the same ex-ante accuracy $q \in (\frac{1}{2}, 1]$ compared to just choosing option X is strictly positive:

$$0.75(1-q)(0-0.25) + 0.25q(1-0.25) - C_\mu(1-q, q) > 0,$$

which creates a contradiction.

Now consider an ex-post accuracy model and assume the posterior separable cost function c does not change when μ does. This paper studies a very general version of the ex-post model, and does not assume that the posterior separable cost function c is strictly convex as many in the literature do. Because of this, it is essentially impossible to reject the model with only a change in belief. If one considers a change in price and a change in belief, however, then it is not hard to reject the ex-post model. For instance, if the big dot is green and $p = 0.25$, then $\Pr(\varpi|\mathrm{Y}, p) = 0.829$, which is significantly higher than 0.75 at the 1% level. But, if the big dot is red and $p = 0.5$, then $\Pr(\varpi|\mathrm{Y}, p) = 0.717$, which is significantly lower than 0.829 at the 1% level. This contradicts what is possible with an ex-post accuracy model when c remains constant as μ changes, and is outlined in Table 5.

What is learned from the comparative statics in this subsection? The ex-ante model once again impose too much structure on the region information outcomes can be observed. When the ex-post model is used it might be necessary to allow the posterior separable cost function c to change when belief changes, as is possible with the aggregate version of the ex-post model.

Table 5: Ex-post Accuracy Model, Changes in Price and Belief

	Big Dot Green	Relationship	Big Dot Red
Prediction	$\Pr(\varpi\|\mathrm{Y}, p = 0.25)$	$\leq$	$\Pr(\varpi\|\mathrm{Y}, p = 0.5)$
Data	0.829	$>^{**}$	0.717
Note: ** indicates rejection with $p < 0.01$ (two-sided Fisher Exact test)			

4.4 Aggregate Ex-ante Accuracy Model: Data

There has not yet been a discussion of an aggregate version of the ex-ante model. As it turns out, the data rejects an aggregate version of the ex-ante model because there is evidence that when subjects choose to 'get more information' they conduct multiple counts of the small dots based on how initial counts compare to what they were expecting. This allows subjects to achieve ex-ante accuracies that differ, even in decision problems where they choose to 'get more information.' As is implied by following proposition, this is inconsistent with an aggregate version of the ex-ante model.

Proposition 9. Given p and μ, if each time the agent chooses to 'get more information' they select one of a finite number of symmetric ex-ante accuracies $q \in (\frac{1}{2}, 1]$, so $\Pr(\mathrm{Y}|\overline{\omega}, p) = \Pr(\mathrm{X}|\underline{\omega}, p) = q$, and the distribution of q they select is independent of the realization of ω, then if I only consider decision problems in which agents choose to 'get more information,' I should observe:

$$\Pr(\mathrm{Y}|\overline{\omega}, p) = \Pr(\mathrm{X}|\underline{\omega}, p).$$

Proof. Trivial.

When $p = 0.25$, and the big dot is green, however, $\Pr(\mathrm{Y}|\overline{\omega}, p)$ is statistically significantly more than $\Pr(\mathrm{X}|\underline{\omega}, p)$ at the 1% level, as it outlined in Table 6. This contradicts the predictions of Proposition 9.

How could this be? It is possible if agents are counting the small dots multiple times, and in particular are counting again if their initial count does not re-affirm their prior belief about which state is more likely. When I examine the data I find evidence that if $p = 0.25$,[12] then subjects spent statistically significantly more time counting when the realized value was not what the subject would have anticipated based on their prior. See Table 7 and Table 8.

In Table 7 and Table 8 the amount of time between a subject selecting 'get more information' and eventually selecting either option X or option Y in a decision problem, which I call **counting time**, is studied with a linear regression to determine how this time changes when the decision problem number and realized value change.

Because subjects were not in a controlled environment when they did the experiment it is possible that there were distractions and based on Figure 8 it seems likely that there were. To get a sense if the results are driven by outliers the left column in each table only considers decision problems in which the subject had no more than 90 seconds of counting time. The 90 second cut-off is fairly arbitrary, but selecting other cut-offs that are similar produce similar results. As can been seen in Figure 8, this cut-off does not remove a large proportion of decision problems. In the case

[12]More than 85% of the data has $p = 0.25$, so statistical power is increased when I consider $p = 0.25$ relative to $p = 0.5$.

Table 6: Aggregate Ex-ante Accuracy Model (Big Dot Green, $p = 0.25$)

Prediction	$\Pr(\text{Y}\vert\overline{\omega},\, p)$	$=$	$\Pr(\text{X}\vert\underline{\omega},\, p)$
Decision Problems with 'get more information'	0.964	$>^{**}$	0.905
Counting Time >= 15 secs.	0.977	$>^{**}$	0.944
Note: ** indicates rejection with $p < 0.01$ (two-sided Fisher Exact test)			

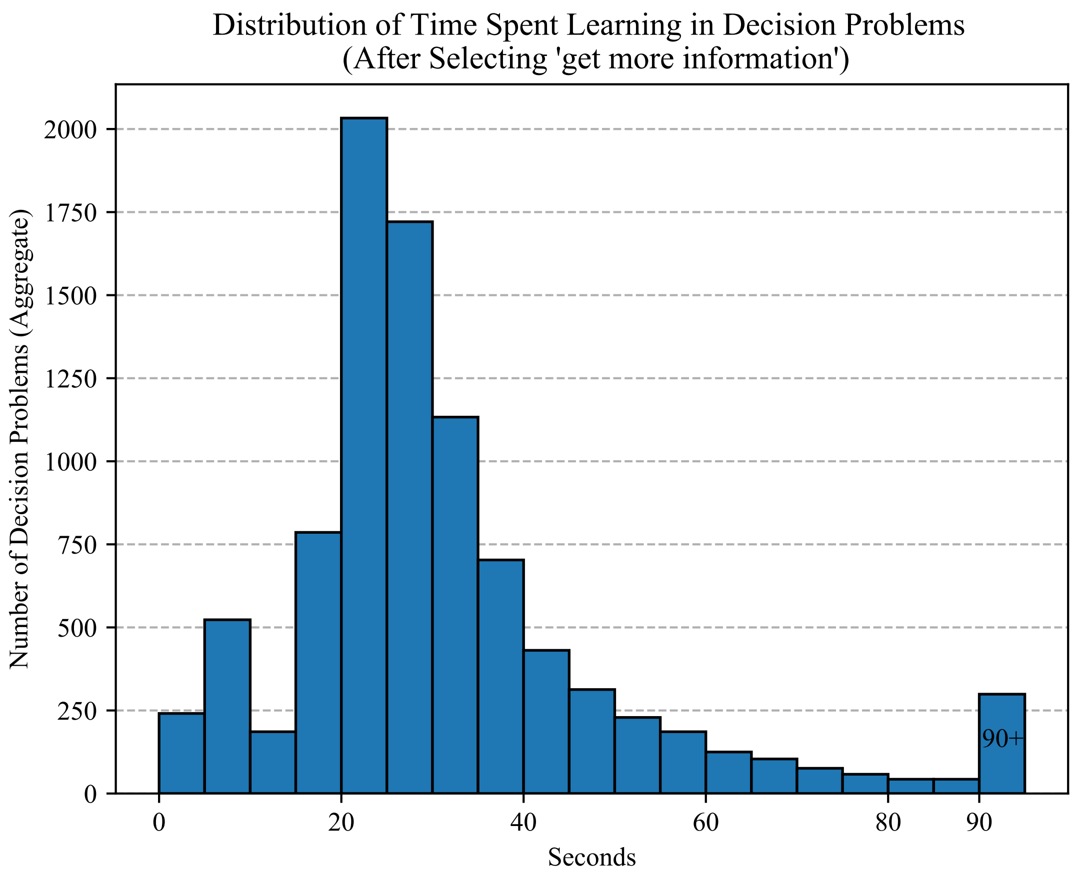
Table 7: Counting Time After 'get more information' (OLS, Big Dot Green, $p = 0.25$)

	Counting Time <= 90 secs.	All Decision Problems
Constant	32.513**	35.627**
	(0.643)	(0.866)
Decision Problem Number	−0.061**	−0.08**
	(0.012)	(0.016)
Payoff from option Y = 0	2.054*	1.622
	(0.812)	(1.099)
Decision Problems (N)	1732	1778
Note: * indicates $p < 0.05$, ** indicates $p < 0.01$		

Table 8: Counting Time After 'get more information' (OLS, Big Dot Red, $p = 0.25$)

	Counting Time <= 90 secs.	All Decision Problems
Constant	34.498**	38.579**
	(0.502)	(0.682)
Decision Problem Number	−0.099**	−0.128**
	(0.007)	(0.009)
Payoff from option Y = 0	−1.726**	−1.91**
	(0.470)	(0.641)
Decision Problems (N)	6169	6383
Note: ** indicates $p < 0.01$		

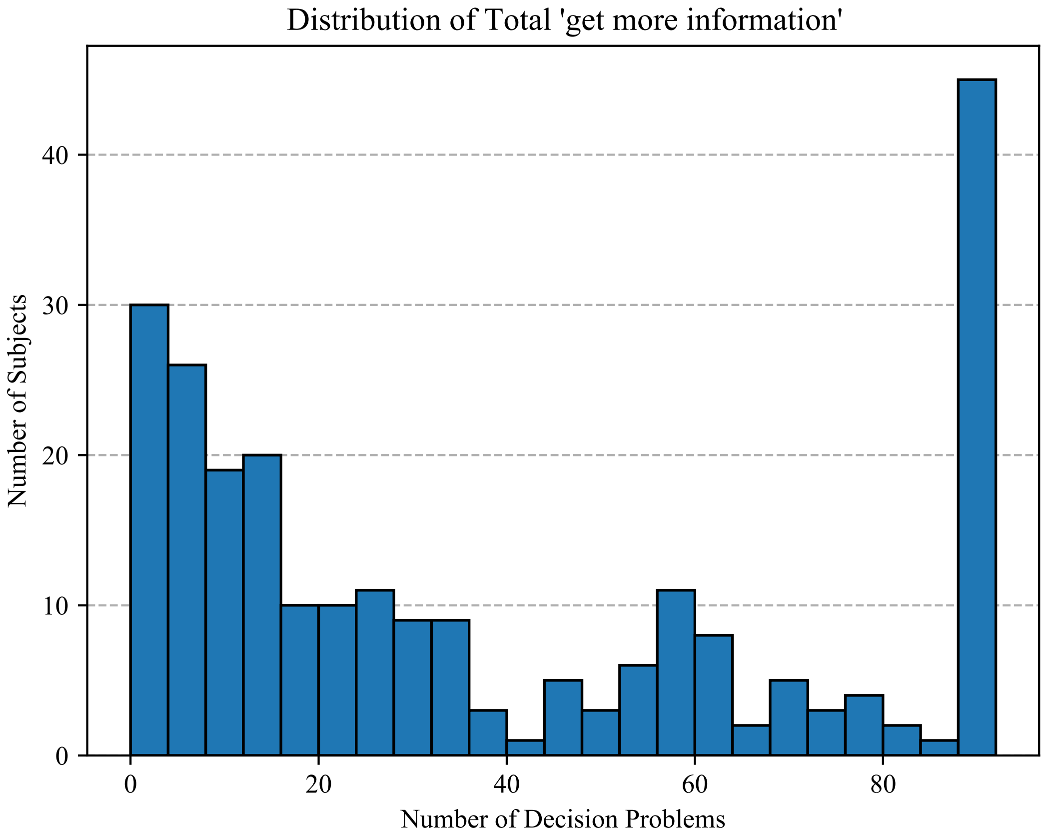
Figure 8:



of a big red dot the imposition of the 90 second cut-off is quite inconsequential. In the case of a big green dot, where there are only about a quarter of the number of decision problems in which subjects choose to ‘get more information,’ the coefficient representing the value of option Y is only significant if the 90 second cut-off is imposed.

Figure 8 also seems to indicate that there may have been decision problems in which the agent selected ‘get more information’ accidentally. So, when ex-ante accuracy is examined in Table 6, one could worry about the result being generated by these accidents, and to address this a row is included that only considers decision problems in which the subject had at least 15 seconds of counting time. The 15 second cut-off is fairly arbitrary, but again selecting other cut-offs that are similar produce similar results. As can been seen in Figure 8, the 15 second cut-off does not remove a large proportion of decision problems, and as can be seen in Table 6 the cut-off does not impact the statistical significance of the finding.

Figure 9:



4.5 Subject Level Analysis: Heterogeneity

As was shown in Section 2, heterogeneity of agents is significant for predicting behavior when price changes, even if each agent behaves in line with a particular model, whether it be the ex-ante accuracy model or the ex-post accuracy model. How much evidence of heterogeneity exists in the experiment data? When different subjects are compared to each other substantial differences can be observed. The easiest way of demonstrating this is to examine the decision problems in which subjects chose to 'get more information.' Figure 9 depicts the number of decision problems subjects chose to 'get more information' in. It is easy to see in Figure 9 that there is a lot of heterogeneity across subjects in terms of the number of times they chose to 'get more information' in a decision problem.

Subjects do differ in their incentive level. The prize that a subject can win at the end of the experiment is based on their treatment group, and is either \$20, \$25, or \$30.[13] This variation,

[13] Subjects do not know that the size of the prize varies for different subjects.

however, does not seem to be the cause of the variation in the number of decision problems subjects learned in. In fact, based on the number of times they chose to 'get more information' on average it was as if the group of subjects with the highest prize had the lowest incentive to learn. The group with the \$30 prize chose to 'get more information' in less rounds on average compared to the other prize groups. Table 9 outlines how the number of decision problems subjects chose to 'get more information' in varies with the prize.

Table 9: Different Prizes and 'get more information' (GMI) Data

	Prize of \$20	Prize of \$25	Prize of \$30
Total Number of Subjects	76	87	80
Average Number of Times Subjects Chose GMI	36.8/92	41.8/92	35.1/92
Median Number of Times Subjects Chose GMI	22.5/92	28/92	26/92
Subjects That Chose GMI in All Decision Problems	12	13	11
Subjects That Chose GMI in Zero Decision Problems	2	2	4

Remember, the incentive to learn varies when price changes (see Section 3.1), thus a subject cannot be indifferent between learning and not learning in every decision problem if, given a belief, they have a fixed cost function for information outcomes. If variation in the size of the prize is not driving the variation in the number of decision problems in which subjects choose to 'get more information,' as depicted in Figure 9, then behavior is indicative of subjects having different cost functions for information. Further, the variation in the cost functions for information outcomes is large relative to the variation in the size of the prize, which is saying something since in a sense the group of subjects with a prize of \$30 has 50% more incentive to learn compared to the group of subjects with a prize of \$20.

Are the choices of a single subject consistent with a single cost function for information outcomes? Are they consistent with a cost function for information outcomes that changes symmetrically when belief changes?

As is shown in Section 3.1, if in each decision problem a subject has a single cost function for information outcomes, then if the big dot is red they have more incentive to learn if the price is $p = 0.25$. Thus, if a subject chooses to 'get more information' in a round with a big dot that is red and $p = 0.5$, they should choose to 'get more information' in all decision problems with a big dot that is red and $p = 0.25$. But in decision problems with a big red dot there are 128 subjects that chose to 'get more information' when $p = 0.5$ and did not choose to 'get more information' when

$p = 0.25$.

It is possible subjects made mistakes when selecting whether or not to 'get more information,' so as a robustness check one can consider subjects that, in decision problems with a big red dot, chose to 'get more information' and waited at least 15 seconds before subsequently selecting option X or option Y when $p = 0.5$ and did not choose to 'get more information' in a decision problem with $p = 0.25$ at least twice. There are still 118 such subjects.

Similarly, as shown in Section 3.1, if in each decision problem a subject has a single cost function for information outcomes, then if the big dot is green they have more incentive to learn if the price is $p = 0.5$. Thus, if a subject chooses to 'get more information' in a round with a big dot that is green and $p = 0.25$, they should choose to 'get more information' in all decision problems with a big dot that is green and $p = 0.5$. But in decision problems with a big green dot there are 91 subjects that chose to 'get more information' when $p = 0.25$ and did not choose to 'get more information' when $p = 0.5$.

Again, it is possible subjects made mistakes when selecting whether or not to 'get more information,' so as a robustness check one can consider subjects that, in decision problems with a big green dot, chose to 'get more information' and waited at least 15 seconds before subsequently selecting option X or option Y when $p = 0.25$ and did not choose to 'get more information' in a decision problem with $p = 0.5$ at least twice. There are still 58 such subjects.

This all means that even without imposing the symmetry from Proposition 8, there is significant evidence that, fixing a belief, subjects do not behave as if they have a single cost function for information outcomes. In total, 179 subjects violated the predictions of Proposition 7, and since 36 subjects chose to 'get more information' in 92 decision problems, and 8 subjects chose to 'get more information' in 0 decision problems, this means 90% of subjects that varied whether or not they chose to 'get more information' from decision problem to decision problem violated the predictions of Proposition 7. What if instead the robustness checks of the proceeding paragraphs are imposed? Still 152 subjects violate the predictions of Proposition 7. Thus, even when examining a single subject, heterogeneity of the cost function seems significant.

4.6 Subject Level Analysis: Fatigue

In the previous subsection it was demonstrated that it frequently seems as if a subject's cost functions for information outcomes changes from decision problem to decision problem. If subjects fatigue as they progress through the decision problems then this would make sense. Can fatigue explain the violations of Proposition 7 discussed in the previous subsection? The short answer is no.

If a subject fatigues, then choosing to 'get more information' should be less appealing. This could be modelled by changing C_μ as the subject progresses through the decision problems. Let C_μ^r denote the cost function of a subject when their belief is μ and they are facing their r^{th} decision problem. A subject's cost function for information outcomes is said to be **consistent with fatigue** if, given belief μ, and integer $t > 0$, $\forall s \in S$ with $\underline{s} \leq \overline{s}$: $C_\mu^r(\underline{s}, \overline{s}) \leq C_\mu^{r+t}(\underline{s}, \overline{s})$.

If the big dots is red, a subject's cost function for information outcomes is consistent with fatigue, and there is a decision problem with $p = 0.25$ in which the subject chooses not to 'get more information,' then in all subsequent decision problems with a big red dot and $p = 0.5$ the subject should choose not to 'get more information.' But, the same 128 subjects from the previous subsection violate this prediction.

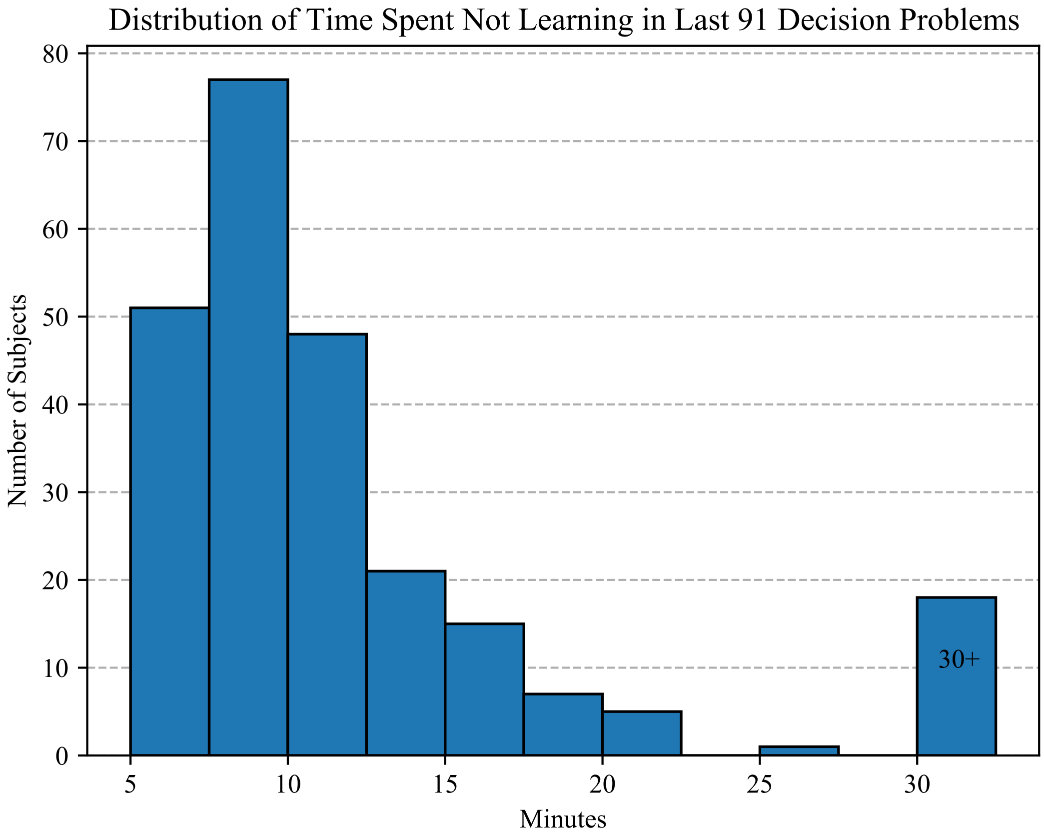
If the big dots is green, a subject's cost function for information outcomes is consistent with fatigue, and there is a decision problem with a big green dot and $p = 0.5$ in which the subject chooses not to 'get more information,' then in all subsequent decision problems with a big green dot and $p = 0.25$ the subject should choose not to 'get more information.' But, 36 subjects violate this prediction.

In total, 143 subjects violate at least one of the predictions from the proceeding two paragraphs, which means 71.9% of subjects that varied whether or not they chose to 'get more information' from decision problem to decision problem violated the predictions of Proposition 7, even if the model allows for a cost function for information outcomes that is consistent with monotonic fatigue. Further, if the the robustness check from the previous subjection are imposed there are still 126 subjects that violate the predictions of the previous 2 paragraphs.

Where then is the variation in the cost function coming from? The notion of fatigue defined in this section does not allow for fatigue to subside. Maybe subjects are taking breaks, however, and their fatigue is periodically alleviated. Figure 10 indicates that subjects are spending substantial time not on screens where they could be view small dots. In the last 91 decision problems all the time before a subject chose to 'get more information' was added together, and the totals of subjects that spent certain amounts of time not learning is displayed in Figure 10. It seems that all subjects took some breaks.

So, behavior is consistent with a subject's cost function for information outcomes changing from decision problem to decision problem, even when belief is controlled for. This change cannot

Figure 10:



be explained with monotonic fatigue, but when the amount of time each individual spends not learning in decision problems is considered, this seems to make sense.[14]

5 Literature Review

This paper focuses on inattention in a specific context where the agent is trying to differentiate between binary outcomes. This simplifying assumption aids tractability, but one can also argue that focusing on binary questions is merited given recent work in the literatures on psychology and psychophysics. When agents are deciding between options eye tracking analysis shows that they successively compare pairs of the options along a single attribute dimension (Noguchi & Stewart, 2014, 2018). This suggests that agents are breaking their learning into a number of smaller queries,

[14]Controlling for a subject's breaks in a sophisticated manner is difficult because they can also take breaks after choosing to 'get more information.' Based on Figure 8, this seems likely as there are over 250 decision problems in which the subject spent at least 90 seconds in the decision problem after choosing to 'get more information.'

and understanding how accurately agents answer these simply queries could be significant. Further, findings in the field of psychophysics suggest that agents are good at discriminating stimuli, but are not good at determining the magnitude of the same stimuli (Stewart, Chater, & Brown, 2006). As a result, in the psychology literature pairwise comparisons are frequently modelled as ordinal in nature (Noguchi & Stewart, 2018), equivalent to questions with binary outcomes. This means agents try to learn if option X is better than option Y in dimension Z, instead of trying to determine how much better is option X than option Y in dimension Z.

To better understand the relationship between the cost of learning and agent behavior, a number of papers have studied axiomatic models of inattention. Different papers choose to focus their axioms on different aspects of the choice environment, however. Caplin et al. (2017), for instance, develop axioms that focus on the choice behavior of an agent after they expend effort to learn about the state of the world. In contrast, de Oliveira (2014) and de Oliveira, Denti, Mihm, and Ozbek (2017) develop axioms that focus on an agent's preferences over choice menus before they expend effort to learn about the state of the world. Ellis (2018) also features axioms that focus on choice behavior, but further assumes that the agent learns by picking a partition of the state space. These papers aim to understand what rational agent behavior tells us about the form of the information cost function. This paper, in contrast, aims to understand what the form of the information cost function tells us about rational agent behavior.

More similar to the theoretical approach of this paper, Pomatto et al. (2019) and Walker-Jones (2020) focus directly on the costs of information. Unlike this paper, both works focus on particular subsets of Posterior Separable cost functions.[15] Placing structure on costs for information is interesting because intuitive properties for costs can lead to unintuitive agent behavior that may be compelling given real-world observations (Gigerenzer & Todd, 1999), but may be mistaken for irrational if structure is instead imposed on behavior.

The paper that is perhaps closest in motivation to the theory work in this paper is the work of Caplin and Dean (2015). They are also interested in determining if choice data can be rationalized with inattention, and study a more general[16] version of the environment this paper discusses in Section 2.1. Caplin and Dean (2015) provide two testable conditions which, given a belief and a utility function, are satisfied if and only if the data can be rationalized with a costly learning model. So, given behavior s and values $u(\underline{\omega})$ and $u(\overline{\omega})$, their result allows us to test whether or not the

[15] Pomatto et al. (2019) actually rule out perfect learning, so their cost function maps onto the extended reals.

[16] Caplin and Dean (2015) allow for any finte Ω and more than two options.

behavior is rationalized by a costly learning model. The result from this paper's Section 2.1, in contrast, studies a setting where only belief is known, and provides conditions that characterize the set of observed behaviors that can be rationalized by a costly learning model when an appropriate pair of values is selected. These distinctions are important because qualitative descriptions of the set of behavior that can be rationalized, like 'the probability of selecting option Y weakly decreases when price increases,' are helpful for making predictions, and the preferences of the agent are only rarely known.

This is not the first paper to note the differences between ex-ante and ex-post modelling techniques (Gentzkow & Kamenica, 2014; Mensch, 2018; Nimark & Sundaresan, 2019; Denti, Marinacci, Rustichini, et al., 2020). For instance, Denti et al. (2020) point out the dichotomi between the two techniques, and identify the set of ex-post cost functions that can be represented with an ex-ante model. The other papers that mention this dichotomy are quite different in their scope, however, and do not provide any characterization of the behavior that can be rationalized by different costly learning models when price changes.

There are a number of papers that experimentally test the implications of models of costly learning in settings with a small number of options that are difficult to rank. Ex-post accuracy models in particular have received a lot of attention. The experiment in this paper is inspired by the experiments conducted by Dean and Neligh (2018), but in their paper they do not have a change in parameters that is equivalent to the change in price that is the primary focus of this paper, and do not observe the outcome of the subjects' decision to 'get more information.' Being able to observe if a subject chooses to 'get more information' is essential for identifying which model the learning is most consistent with. Dewan and Neligh (2020) study a different set of costly learning tasks experimentally and model the agent in a 'Task Based' fashion that is similar to this paper's ex-ante model,[17] but again they do not observe the decision to 'get more information,' and do not have a change in parameters that is equivalent to a change in price. When these papers change option values they primarily do so in a multiplicative fashion, i.e. they do something analogous to multiplying p and ω by a constant so incentive to learn is unambiguously higher or unambiguously lower. Understanding how inattention changes when price changes is important because changes in price are so commonly observed in the real world, whereas changes in belief or a multiplicative change in option values are harder to come by. Changes in price are also slightly more complicated

[17]Interestingly enough, in the context Dewan and Neligh (2020) study, the Task Based model and the Posterior Separable model are not necessarily contradictory, and one can, for instance, define a Task Based model that is behaviorally equivalent to the commonly applied Posterior Separable model of Shannon Entropy.

to analyse because it can be unclear whether or not learning costs should increase or decrease when price changes.

Ambuehl (2017) and Ambuehl, Ockenfels, and Stewart (2020) experimentally and theoretically explore environments with changes in parameters that are equivalent to changes in price, but they do not feature changes in belief, and do not observe the decision to 'get more information.' On the theory side, (Ambuehl et al., 2020) study the special case of ex-post accuracy models described by Shannon Entropy (Shannon, 1948; Matějka & McKay, 2015), while (Ambuehl, 2017) studies a slightly less general version of the ex-post model compared to this paper, and establishes the necessity of the conditions in Proposition 4, but not sufficiency.

There is a growing body of literature that demonstrates the importance of inattention and choice mistakes in standard economic settings. Chetty, Looney, and Kroft (2009), for instance, show that including sales tax in posted prices reduces demand. This is presumably because consumers were underestimating sales tax when making purchasing decisions, and were buying things that they would have been better off not purchasing. At first glance this simple example may not seem significant to other economic problems, but the presence of inattention in such a standard setting, where it seems like information should be costless to obtain, demonstrates how pervasive inattention is. In recent years papers have identified the significance of inattention in a wide variety of fields, ranging from finance (Huberman, 2001), to labor search (Acharya & Wee, 2019), to trade (Dasgupta & Mondria, 2018), to voting behavior (Shue & Luttmer, 2009).

6 Conclusion

I both theoretically and experimentally study choice in a setting with costly information acquisition. My model and experiment focus on a simple context where an agent has two options, a safe option with a known value, and an uncertain option with an unknown value.

In a revealed preference setting I characterize the choice behavior patterns that are consistent with price changes and models that measure mistakes in both an ex-ante 'Task Based' (Dewan & Neligh, 2020) fashion, and an ex-post 'Posterior Separable' (Caplin et al., 2017) fashion. I then discuss the implications of the models for datasets that aggregate choices across individuals who might have heterogeneous private pieces of information and heterogeneous costs for achieving a given low probability of a choice mistake. I fully characterize the mistake patterns that are consistent with the resultant aggregate version of the ex-post model.

Using a novel design that allows me to observe if subjects tried to learn in each decision problem, I evaluate the predictions of the theoretical models experimentally in a setting where the researcher can observe choice mistakes that should be attributed to the cost of acquiring information. Observing whether subjects attempt to acquire information results in a clean identification of what information they acquire on average when they do learn, since there is no need to average choice mistakes over instances where subjects do and do not try to acquire information.

The experimental results indicate that subjects are sophisticated when choosing when to learn and what to learn. Subjects are most likely to 'get more information' at the combination of price and belief that theoretically provide the highest incentive to learn according to both the ex-ante and ex-post models, and there is evidence that they accumulate information gradually in a way that allows them to achieve ex-ante accuracies that differ depending on the realized payoff from the uncertain option. When the realized payoff from the uncertain option does not align with the payoff that was most likely according to a subject's prior belief, they spend more time acquiring information. This suggests that subjects are choosing to learn in a way they are aware could lead to errors, and learning more if initial information is unexpected. As a result, subjects achieve ex-ante accuracies that differ in a statistically significant fashion, which would be more in line with the ex-post model. This is true even though subjects could presumably learn as accurately as they want to with a single count.

When price changes the standard ex-post model does better than the ex-ante model predicting how choices change at the aggregate level, but does poorly when both price and prior belief change. This all suggests that subjects are more flexible when choosing what information to acquire than the simple ex-ante model allows, and that understanding the heterogeneity of subjects is significant for understanding how choices changes when prices change.

References

Acharya, S., & Wee, S. L. (2019). Rational inattention in hiring decisions. *FRB of New York Staff Report*(878).

Allais, M. (1953). Le comportement de l'homme rationnel devant le risque: critique des postulats et axiomes de l'école américaine. *Econometrica: Journal of the Econometric Society*, 503–546.

Ambuehl, S. (2017). An offer you can't refuse? incentives change how we inform ourselves and what we believe. *CESifo Working Papers*(6296).

Ambuehl, S., Ockenfels, A., & Stewart, C. (2020). Who opts in?

Azrieli, Y., Chambers, C. P., & Healy, P. J. (2018). Incentives in experiments: A theoretical analysis. *Journal of Political Economy*, *126*(4), 1472–1503.

Azrieli, Y., Chambers, C. P., & Healy, P. J. (2020). Incentives in experiments with objective lotteries. *Experimental Economics*, *23*(1), 1–29.

Blackwell, D. (1953). Equivalent comparisons of experiments. *The annals of mathematical statistics*, 265–272.

Caplin, A., & Dean, M. (2015). Revealed preference, rational inattention, and costly information acquisition. *American Economic Review*, *105*(7), 2183–2203.

Caplin, A., Dean, M., & Leahy, J. (2017). *Rationally inattentive behavior: Characterizing and generalizing shannon entropy* (Tech. Rep.). National Bureau of Economic Research.

Chen, D. L., Schonger, M., & Wickens, C. (2016). otree—an open-source platform for laboratory, online, and field experiments. *Journal of Behavioral and Experimental Finance*, *9*, 88–97.

Chetty, R., Looney, A., & Kroft, K. (2009). Salience and taxation: Theory and evidence. *American economic review*, *99*(4), 1145–77.

Colombo, L., Femminis, G., & Pavan, A. (2014). Information acquisition and welfare. *The Review of Economic Studies*, *81*(4), 1438–1483.

Dasgupta, K., & Mondria, J. (2018). Inattentive importers. *Journal of International Economics*, *112*, 150–165.

Dean, M., & Neligh, N. L. (2018). Experimental tests of rational inattention.

Denti, T., Marinacci, M., Rustichini, A., et al. (2020). *Experimental cost of information* (Tech. Rep.).

de Oliveira, H. (2014). *Axiomatic foundations for entropic costs of attention* (Tech. Rep.). Mimeo.

de Oliveira, H., Denti, T., Mihm, M., & Ozbek, K. (2017). Rationally inattentive preferences and hidden information costs. *Theoretical Economics*, *12*(2), 621–654.

Dewan, A., & Neligh, N. (2020). Estimating information cost functions in models of rational inattention. *Journal of Economic Theory*, *187*(105011).

Ellis, A. (2018). Foundations for optimal inattention. *Journal of Economic Theory*, *173*, 56–94.

Gentzkow, M., & Kamenica, E. (2014). Costly persuasion. *American Economic Review*, *104*(5), 457–62.

Gigerenzer, G., & Todd, P. M. (1999). Fast and frugal heuristics: The adaptive toolbox. In *Simple heuristics that make us smart* (pp. 3–34). Oxford University Press.

Greiner, B. (2015). Subject pool recruitment procedures: organizing experiments with orsee. *Journal of the Economic Science Association*, *1*(1), 114–125.

Hébert, B., & Woodford, M. (2017). *Rational inattention and sequential information sampling* (Tech. Rep.). National Bureau of Economic Research.

Huberman, G. (2001). Familiarity breeds investment. *The Review of Financial Studies*, *14*(3), 659–680.

Matějka, F., & McKay, A. (2015). Rational inattention to discrete choices: A new foundation for the multinomial logit model. *American Economic Review*, *105*(1), 272–98.

Mensch, J. (2018). Cardinal representations of information. *Available at SSRN 3148954*.

Morris, S., & Strack, P. (2019). The wald problem and the relation of sequential sampling and ex-ante information costs.

Nimark, K. P., & Sundaresan, S. (2019). Inattention and belief polarization. *Journal of Economic Theory*, *180*, 203–228.

Noguchi, T., & Stewart, N. (2014). In the attraction, compromise, and similarity effects, alternatives are repeatedly compared in pairs on single dimensions. *Cognition*, *132*(1), 44–56.

Noguchi, T., & Stewart, N. (2018). Multialternative decision by sampling: A model of decision making constrained by process data. *Psychological review*, *125*(4), 512.

Pomatto, L., Strack, P., & Tamuz, O. (2019). The cost of information.

Shannon, C. E. (1948). A mathematical theory of communication. *The Bell System Technical Journal*, *27*(3), 379–423.

Shue, K., & Luttmer, E. F. (2009). Who misvotes? the effect of differential cognition costs on election outcomes. *American Economic Journal: Economic Policy*, *1*(1), 229–57.

Sims, C. A. (2003). Implications of rational inattention. *Journal of monetary Economics*, *50*(3), 665–690.

Singular "they". (n.d.). Retrieved 2020-11-17, from https://apastyle.apa.org/style-grammar-guidelines/grammar/singular-they

Stewart, N., Chater, N., & Brown, G. D. (2006). Decision by sampling. *Cognitive psychology*, *53*(1), 1–26.

Verrecchia, R. E. (1982). Information acquisition in a noisy rational expectations economy. *Econometrica: Journal of the Econometric Society*, 1415–1430.

Wald, A. (1945). Sequential tests of statistical hypotheses. *The annals of mathematical statistics*, *16*(2), 117–186.

Walker-Jones, D. (2020, October). *Rational inattention and perceptual distance.* Retrieved from https://www.dwalkerjones.com

Appendix 1: The Experiment

This appendix complements Section 3. There is some repetition to make things more clear, but the explanation of the experiment in this appendix is not complete in an of itself.

Participants were recruited for the experiment using ORSEE (Greiner, 2015) from the Toronto Experimental Economics Laboratory recruitment pool. Subjects signed up ahead of time for a particular day, either the 21st, 22nd, 23rd, 28th, 29th, or 30th of September 2020.[18] Subjects were told ahead of time they would be sent a link at 8 AM EDT, and would have until 8 PM EDT to finish the experiment.[19] In total 270 undergraduate students from the University of Toronto attempted to partake in the experiment, and 243 of them completed the experiment.[20] I refer to the 243 students that completed the experiment at the 'subjects.'

Each subject began by consenting to participate in the experiment. They then saw a welcome screen that explained that they were about to be trained and quizzed. They were told: "Please read the following instructions carefully. Understanding what is going on will help you earn more money. You will first be trained, and then you will be quizzed, to make sure you understand how everything works. There will be 10 questions in the quiz. You must answer each question correctly before you can move on to the next question. When you complete the quiz you will earn the \$5 training payment. In addition, each quiz question you answer correctly on the first try earns you another \$0.5 training bonus payment, so you can earn up to \$10 just by doing well during the training." I wanted to incentivize students to pay attention during the training so that they understood the environment and their choices would be indicative of their preferences. All payments were in Canadian dollars.

The training consisted of three pages, which are figures 11, 12, and 13, below. In the second training page subjects were required to select 'get more information' so that they knew what happened if they did. If they did not, then they got an error message asking them to do so. The 10 quiz questions can be seen at the end of this appendix. If a subject got a quiz question wrong the

[18] I had 24 subjects on the 21st, 39 on the 22nd, 48 on the 23rd, 43 on the 28th, 46 on the 29th, and 70 on the 30th.

[19] On the 22nd the session was extended until 9 PM EDT because students experienced crashes. Then on September 28th the Economic Department's servers were unexpectedly down until almost 10 AM EDT, so students were given until 10 PM EDT to complete the experiment.

[20] There were 315 undergraduate students that signed up to receive a link for the experiment but 45 of them did not use the link. This proportion is in line with the "turnout" rates of other experiments run with the recruitment pool. A handful of students experienced one of two errors on September 22nd. One seemed to be caused by a server problem, while the other was caused by a missing image file. Four of the students that experienced errors did not want to re-start. Two students started the experiment with less than 20 minutes left before their session finished and could not finish as a result. This left 264 students, but some of them chose to drop out, as is explained later, so 243 ended up completing the experiment.

order of the answers in the drop down menu randomly permuted so that subjects could not just try them in order. The quiz verified that subjects understood the environment, but also gave us another opportunity to train them. In the data I can see how many times each subject attempted each question. I also know how long subjects spent on each page. Among subjects the average time spent on the welcome page, training pages, and quiz pages was 17.9 minutes, and the average quiz score was 7.6/10. So, subjects in general put a lot of effort into understanding the environment, and were quite successful in doing so.

After the quiz subjects faced 100 rounds of 'investment decisions' (the last 92 of which are the decision problems I study in the body of the paper). In each round the subject selects one of two options, option X or option Y. In all 100 rounds the agent selects from their options in a drop down menu, and the order of the options is randomly generated in each round.

The subject earned probability points in each round. Subjects gradually increased their collection of probability points so as to have a higher chance of winning the draw at the end of the 100 rounds. If they won the draw they would receive a monetary prize of either \$20, \$25, \$30. The prize they would receive if they won was determined by the subject's treatment group, and was displayed to the subject throughout the training pages and rounds.[21] In a round the smallest increase the subject can get in their probability of winning the prize is zero percentage points, and the largest increase is one percentage point. In each round option X is worth an amount of percentage points strictly between 0 and 1, and option Y is worth 0 or 1 percentage points.

I paid subjects in probability points so I could attempt within subject analysis in an incentive compatible way. The ideal when it comes to incentive compatibility is that in each of the 100 rounds the subject chooses what they would have chosen if they had only had to make a decision in that one round (Azrieli et al., 2018; Azrieli, Chambers, & Healy, 2020). If the preferences of the subject change while the subject is making decisions, then their choices are not indicative of a single set of preferences, and analysis is more difficult.

In the setting of my experiment if I pay the subject money in each round then their marginal value for money could decrease as the rounds progress (wealth effects). Further, it would be difficult to separate this change in preferences from fatigue. The standard solution is to us a "random incentive system" (RIS), and pay the agent based on their decision in that round (Allais, 1953). In our setting this strategy is not incentive compatible. The agent needs to consider the cost of

[21]Because the subjects participated on-line, they were transfered the money electronically within 24 hours of their session ending.

their learning in the setting studied in this paper, and they cannot defer their learning until they know which round is selected. If I increase the number of rounds they see, and keep the monetary payments the same, their accuracy should go down. The benefit (in expectation) from making a good selection in a round decreases while their cost of learning stays the same.

Further, in contrast with Azrieli et al. (2018), I need the subject to know the probability with which each round is selected, otherwise I inadvertently introduce ambiguity into their optimization problem. So, I require an objective lottery of one form or another, which leaves us susceptible to issues if agents do not reduce compound lotteries.[22]

Even when told each of the 100 rounds is drawn with equal probability the subject might subconsciously think that certain rounds, with say a lower value for the safe option, are more likely. This would be problematic because I would then essentially increase the relative learning costs in rounds with high values for the safe option.

Paying subjects in probability points is mathematically equivalent to RIS if subjects reduce compound lotteries and know each round is selected with equal chance: A problem in one of the first 8 rounds[23] with $p = 0.5$ and a big green dot is equivalent to deciding between a 50% chance of winning the prize and a 75% chance of drawing a 100% chance of winning the prize if there is only a 1 in a 100 chance that decision is selected for payment.

If subjects do not accurately internalize an objective lottery over rounds, then paying in probability points might be better for eliciting preferences from a pedagogical perspective. This strategy is certainly not infallible, however.

The value of a one percent increase in the probability of winning the prize might depend on the subject's current probability of winning the prize. I think this is of particular concern in two cases, when the subjects probability of winning the prize is moving away from 0%, and when it is moving to 100%. I think a probability point should have essentially the same value in two different rounds if neither of these two cases are occurring. This means that if I can move a subjects chance of winning away from 0% and 100% I could reduce certainty effects, while certainty effects would be persistent with RIS.

The first 8 rounds of investment decisions focus on concerns around incentive compatibility. In the first 8 rounds subjects could not 'get more information' and they had to make a decision based on the value of the safe option X and the big dot. The first 8 rounds thus relate to incentive

[22] For instance, a subject may select a 20% chance of 100 probability points over 25 probability points.

[23] In the first 8 rounds subjects cannot 'get more information,' see below.

compatibility in three ways. First, if the subject chooses the safe option in any of the first 8 rounds then they know they cannot achieve a 100 percent chance of winning the prize, and certainty effects are mitigated at least partially. Second, if the subject chooses X in any of the first 8 rounds their probability of winning the prize is strictly above zero, and I do not need to worry about them making a decision later purely because it means they guarantee a chance of winning the prize. Third, it tests whether or not agents are reducing compound lotteries in the rounds, i.e. selecting the option that produces the larger average increase in the chance of winning the prize,[24] in a setting where I know their beliefs, and I do not need to worry about how accurate their learning was before they decided.

In the first 8 rounds each subject saw the same sequence of big dots (which form beliefs) and values for the safe option X (prices). The big dots were green, green, red, red, red, red, red, and red, and the values for option X were 0.8, 0.7, 0.2, 0.5, 0.4, 0.3, 0.24, and 0.26.

The decisions of the subjects in the first 8 rounds indicate that they frequently violate reduction compound lotteries, i.e. subjects did not always select the option that provided the larger average increase in probability points in each of the first 8 rounds. Only 20.6% of subjects chose the options that resulted in the higher average increase in their probability of winning the prize in all of the first 8 rounds, and only 51.9% of subjects did so in at least 6 of the 8 first rounds. In the first 8 rounds, the percent of subjects that chose the option that caused the higher average increase in the chance of winning the prize was: 73.3%, 74.9%, 61.3%, 91.8%, 86%, 77%, 61.7%, and 73.3%. In relation to the other of the first 8 rounds, the behavior in the third round, in which $\mu(1) = \frac{1}{4}$ and $p = 0.2$, is particularly strange, but this is the first transition from a big green dot to a big red dot. Even in an environment where it would seem that a researcher should be able to induce the preferences of subjects, it seems surprisingly difficult.

In each decision problem subjects had the option to stop doing decision problems. They were told that if they decided to 'exit' they would maintain their current probability of winning the prize, and immediately find out if they had won or not, forfeiting the chance to further increase their chance of winning. I gave them the option to leave because in the model agents are assumed to only have two options that are appealing to them, option X and option Y. If an agent fatigues, and they no longer think it is worth their time to select the better option, but they cannot stop without loosing any probability of winning they had accrued, then they might rush through the experiment, selecting options not because they are indicative of the agent's preferences, but because the agent

[24] In each of the 100 rounds the average increase each option provides is displayed next to the option. See Figure 5.

is trying to finish as fast as they can. So, if a subject prefers to stop doing decision problems, I let them, so my data would be less noisy and more indicative of preferences.

I had 21 subjects decide not to finish the experiment, which left us with a sample of 243 subjects. Less than ten percent of students dropping out is not overly concerning since they were participating over the internet and could have had any number of distractions arise.

It was hoped that red and green would have intuitive value to the subjects. The downside with red and green is that about one in twenty people have red/green color blindness. To combat this issue, each dot displayed has a black letter in either R of G, so that subjects can still differentiate between red and green dots even if they are color blind. See Figure 6 for an example. I did not receive any complaints about the dots being hard to differentiate.

Before any subjects participated in the experiment ten different "treatments" of image files were generated ($100 \times 10 = 1000$ rounds of images, each consisting of a big dot image and an image of 100 small dots). This means that 1000 times a big dot color was drawn. The chance of the big dot being green was $\frac{1}{4}$ and the chance of the big dot being red was $\frac{3}{4}$. After the big dot color for the round was determined the composition of the small dots was drawn. In each instance there was a $\frac{3}{4}$ chance that 51 of the small dots would match the color of the big dot and a $\frac{1}{4}$ that 51 of the small dots would not match the color of the big dot. Either way, the order of the small dots was randomly generated given the drawn proportion. After this process was completed, the first eight rounds of images were used for all ten treatments, but other than that the images were not altered. This means, for instance, that if a subject was assigned to image "treatment" nine, their first eight rounds of images were the first eight rounds of images generated, and their last 92 rounds of images were the 809th through 900th rounds of images generated.

Figure 11:

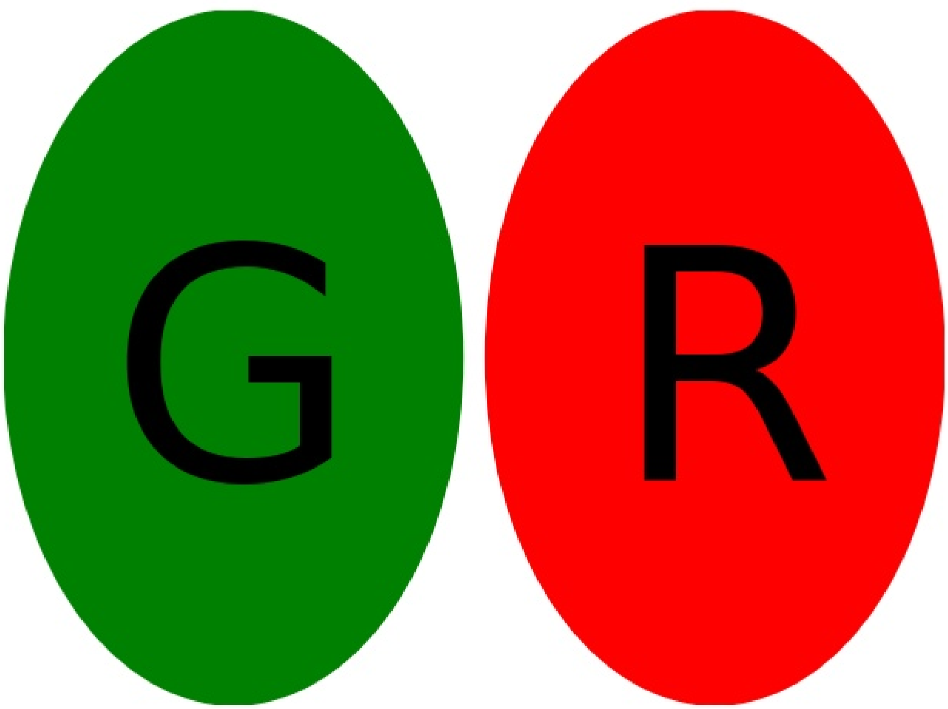
Training (1 of 3):

The experiment consists of 100 rounds of investment decisions, which does not include the training round (Round 0). Each of the 100 rounds provides you with the opportunity to increase your probability of winning a prize of $25 at the end of the experiment. This prize is in addition to whatever you earn during the training process.

You will not be able to increase your probability of winning the prize by more than 1 percentage point in any one round. Ideally, you would thus like to increase your probability of winning the prize by 1 percentage point in each round, so that you win the prize with a 100% probability when the experiment is over, but it is extremely unlikely that this will be possible.

In each investment decision you get to choose between two investment options, option X and option Y.
Option X always increases your probability of winning the prize by a displayed positive amount, which is always less than 1 percentage point.
Option Y, in contrast, either does not increase your probability of winning the prize at all, or it increases your probability of winning the prize by 1 percentage point.

To help provide you with some information about the chance that option Y increases your probability of winning the prize, in each round there is one large dot that is either red with a black R in the middle of it (example below on right), or green with a black G in the middle of it (example below on left).



When the large dot is red, this is a bad sign for the value of option Y, and there is only a 1 out of 4 chance (25% chance) that option Y increases your probability of winning the prize by 1 percentage point, and a 3 out of 4 chance (75% chance) that option Y leaves your probability of winning the prize unchanged. This means that when the large dot is red, on average option Y increases your probability of winning the prize by 0.25 percentage points.

When the large dot is green, this is a good sign for the value of option Y, and there is a 3 out of 4 chance (75% chance) that option Y increases your probability of winning the prize by 1 percentage point, and only a 1 out of 4 chance (25% chance) that option Y leaves your probability of winning the prize unchanged. This means that when the large dot is green, on average option Y increases your probability of winning the prize by 0.75 percentage points.

In each round there is a 3 out of 4 chance (75% chance) the big dot is red, and a 1 out of 4 chance (25% chance) the big dot is green. In the first 8 rounds (rounds 1 through 8) the large dot is the only source of information available to you.

Next

Figure 12:

Training (2 of 3):

In the last 92 rounds (rounds 9 through 100), you can get more information if you choose to. When you choose to 'Get more information,' you are shown 100 small dots, each of which is red with a black R in the middle, or green with a black G in the middle. Together, the small dots tell you for sure whether or not option Y increases your probability of winning the $25 prize by 1 percentage point in the round you are in.

In each round that you can 'Get more information,' there are either 49 small red dots and 51 small green dots, or 51 small red dots and 49 small green dots.
If there are 49 small red dots and 51 small green dots, then option Y increases your probability of winning the prize by 1 percentage point.
If there are 51 small red dots and 49 small green dots, then option Y does not increase your probability of winning the prize.

Under the dotted line below is an example of a round where you can choose to get more information. Notice that the round number is displayed, and the amount that option X increases your probability of winning the prize is displayed.

The large dot is green, so you know there is a 3 out of 4 chance (75% chance) option Y increases your probability of winning the prize by 1 percentage point, but you cannot know for sure unless you choose to 'Get more information.'

Please select 'Get more information' so you see what happens when you do.

Round number: 0 of 100
Option X increases your probability of winning the $25 prize by 0.5 percentage points

Please choose an option:

Remember:

If the large dot is red, then there is a 3 out of 4 chance (75% chance) option Y increase your probability of winning the prize by 0 percentage points, and a 1 out of 4 chance (25% chance) it increases your probability of winning the prize by 1 percentage point. This means that when the large dot is red, on average option Y increases your probability of winning the prize by 0.25 percentage points.

If the large dot is green, then there is a 3 out of 4 chance (75% chance) option Y increases your probability of winning the prize by 1 percentage point, and a 1 out of 4 chance (25% chance) it increases your probability of winning the prize by 0 percentage points. This means that when the large dot is green, on average option Y increases your probability of winning the prize by 0.75 percentage points.

If you choose to 'Get more information,' 100 small dots will appear that you can use to determine how much option Y increases your probability of winning the prize.

Next

Figure 13:

Training (3 of 3):

Under the dotted line below is an example of what you would see after requesting to 'Get more information' in a round. Selecting this option has made the 100 small dots for this round appear.

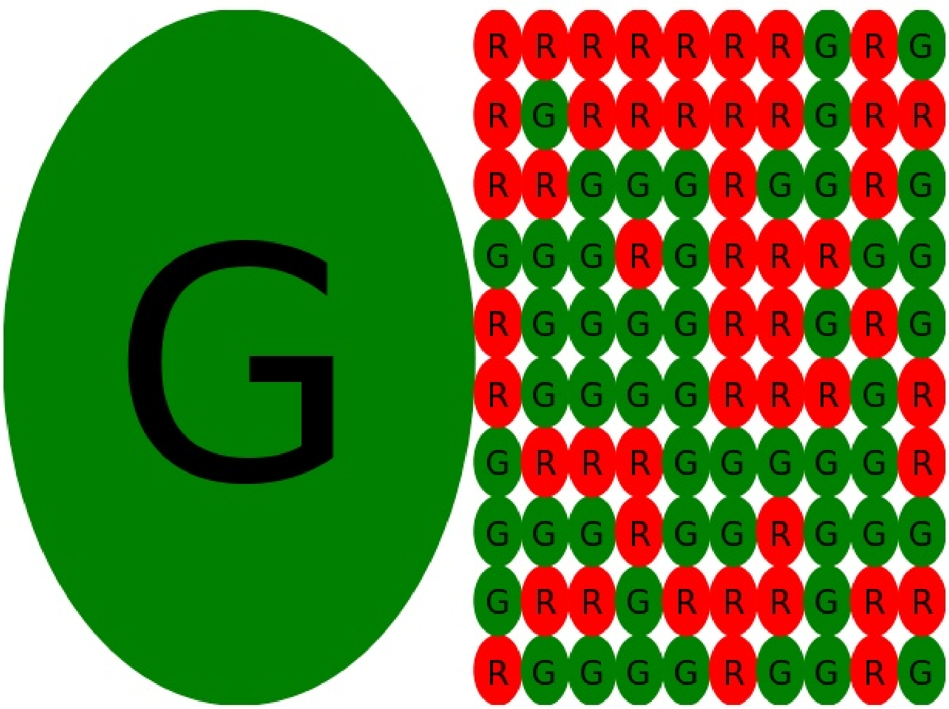
Try counting the number of small red dots and or the number of small green dots below. You should find there are 49 red dots and 51 green dots, which means option Y would increase your probability of winning the prize by 1 percentage point. So in this round, option Y increases your probability of winning the prize by more than option X. This training round (Round number 0) does not count towards your eventual probability of winning the prize.

After you have finished the quiz, if you want to quit the experiment you will be given the option to do so at the bottom of the screen in each round. When you choose to exit, you still have the chance to win the $25 prize, you do not lose any probability you have acquired of winning the prize, but you do lose the chance to further increase your probability of winning the prize.

Round number: 0 of 100

Option X increases your probability of winning the $25 prize by 0.5 percentage points

Please choose an option:



Remember:

In each round, option Y always increases your probability of winning the prize by either 0 or 1 percentage points.

In each round, there are always either 49 small red dots and 51 small green dots, or 51 small red dots and 49 small green dots.

If there are 49 small red dots and 51 small green dots, then option Y increases your probability of winning the prize by 1 percentage point.

If there are 51 small red dots and 49 small green dots, then option Y increases your probability of winning the prize by 0 percentage points.

Next

Quiz time! (Question 1 of 10)

It is now time to do the quiz. Remember, to move on to the next question you must first answer the current question correctly. Completing the quiz earns you the $5 training payment, and in addition to the training payment, you can earn another $0.5 for each question you get correct on your first attempt to answer it.

Which of the following is correct?

Option X sometimes increases your probability of winning the prize by more than 1 percentage point

Sometimes selecting the wrong option can reduce your probability of winning the prize

In each round, option Y always increases your probability of winning the prize by 1 percentage point

In each round, option Y either increases your probability of winning the prize by 1 percentage point, or does not change it

Quiz time! (Question 2 of 10)

If you see the big dot below in a round, which of the following is NOT correct?

If no other information is available then you must be in the first 8 rounds

If you are in the last 92 rounds you can get more information

If no other information is available then there is a 3 out of 4 chance that option Y increases your probability of winning the prize by 1 percentage point

If you are in the first 8 rounds you can get more information

Quiz time! (Question 3 of 10)

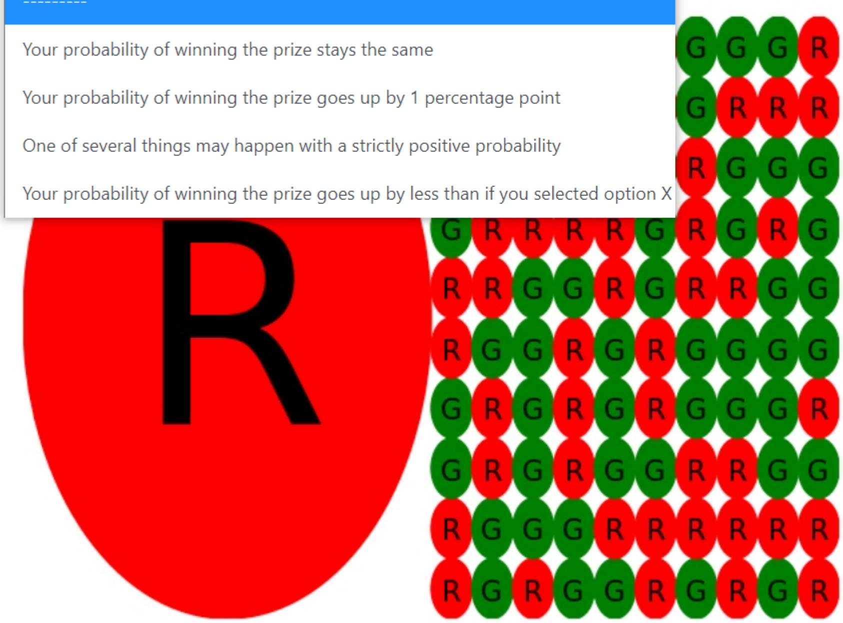
In the last 92 rounds, if you see the the big dot below, if you choose to 'Get more information,' how many small green dots with black Gs will be displayed?

- ---------
- All numbers between 0 and 100 occur with a strictly positive probability
- Either 49 or 51
- 51
- 100

Quiz time! (Question 4 of 10)

In the image below there are 51 small green dots. If you see such an image in a round, what happens when you select option Y?

- ---------
- Your probability of winning the prize stays the same
- Your probability of winning the prize goes up by 1 percentage point
- One of several things may happen with a strictly positive probability
- Your probability of winning the prize goes up by less than if you selected option X



Quiz time! (Question 5 of 10)

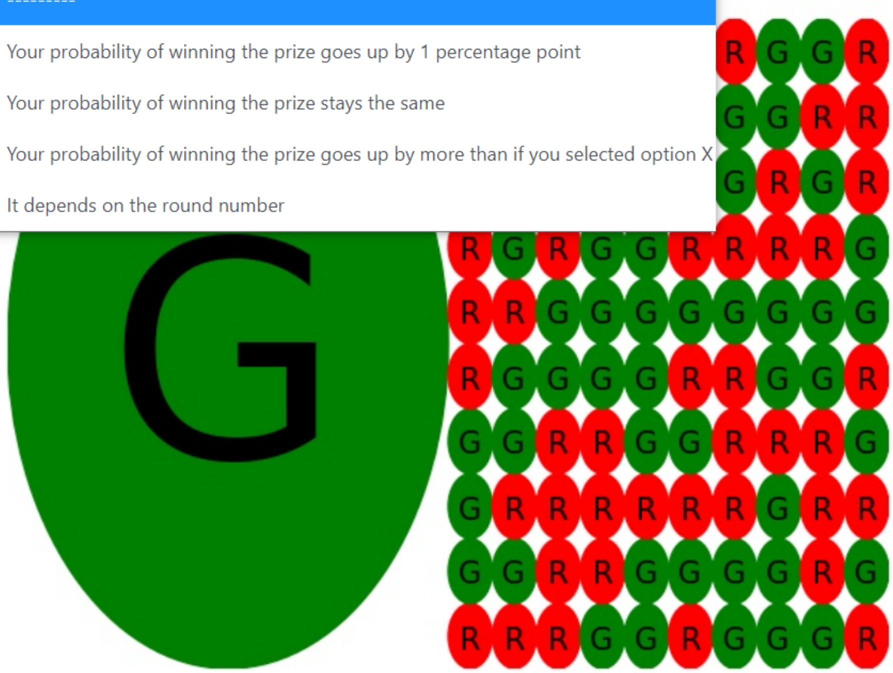
If the big dot below is visible in a round, which of the following is correct?

- ---------
- If you are in the first 8 rounds you can get more information
- If no other information is available then there is a 1 out of 4 chance that option Y increases your probability of winning the prize by 1 percentage point
- If no other information is available then you must be in the last 92 rounds
- If no other information is available then there is a 3 out of 4 chance that option Y increases your probability of winning the prize by 1 percentage point

Quiz time! (Question 6 of 10)

In the image below there are 51 small red dots. If you see such an image in a round, what happens when you select option Y?

- ---------
- Your probability of winning the prize goes up by 1 percentage point
- Your probability of winning the prize stays the same
- Your probability of winning the prize goes up by more than if you selected option X
- It depends on the round number



Quiz time! (Question 7 of 10)

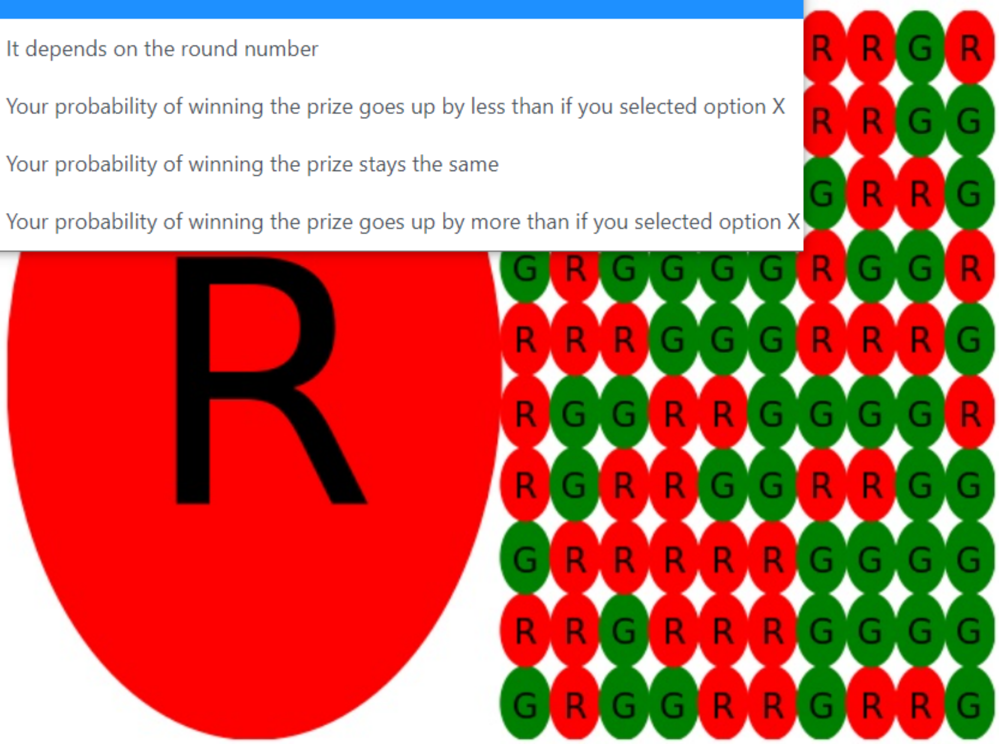
Count the number of small green dots in the image below. If you see such an image in a round, what happens when you select option Y?

It depends on the round number

Your probability of winning the prize goes up by less than if you selected option X

Your probability of winning the prize stays the same

Your probability of winning the prize goes up by more than if you selected option X



Quiz time! (Question 8 of 10)

When you choose to 'Get more information' in a round, which of the following is correct?

Selecting option X in the same round causes your probability of winning the prize to stay the same

100 small dots appear that together tell if option Y increases your probability of winning the prize by 1 percentage point

Your probability of winning the prize goes up by 1 percentage point

Your probability of winning the prize may decrease

Quiz time! (Question 9 of 10)

If the large dot below is the only information available, then which of the following is correct?

On average, option Y increases your probability of winning the prize by 0.25 percentage points

On average, option Y increases your probability of winning the prize by 0 percentage points

On average, option Y increases your probability of winning the prize by 1 percentage point

On average, option Y increases your probability of winning the prize by 0.75 percentage points

Quiz time! (Question 10 of 10)

If the large dot below is the only information available, then which of the following is correct?

On average, option Y increases your probability of winning the prize by 0.25 percentage points

On average, option Y increases your probability of winning the prize by 0.75 percentage points

On average, option Y increases your probability of winning the prize by 0 percentage points

On average, option Y increases your probability of winning the prize by 1 percentage point

Appendix 2: The Proof

Proof of Proposition 1. I begin with the necessity of each of the points. **(i)**: if $\exists p \in \mathcal{P}$ such that $\Pr(\mathrm{Y}|\underline{\omega},\, p) > \Pr(\mathrm{Y}|\overline{\omega},\, p)$, then the behavior is not rationalized by a costly learning model because the agent would have achieved a strictly higher payoff at p by choosing either (depending on the value of $u(\underline{\omega})$ relative to p) $s = (\Pr(\mathrm{Y}|\overline{\omega},\, p),\, \Pr(\mathrm{Y}|\overline{\omega},\, p))$ or $s = (1,\, 1)$).

(ii) : Suppose not, so $p_1 < p_2$, and :

$$\Pr(\mathrm{Y}|p_1) = \mu(\overline{\omega})\Pr(\mathrm{Y}|\overline{\omega},\, p_1) + \mu(\underline{\omega})\Pr(\mathrm{Y}|\underline{\omega},\, p_1)$$

$$< \mu(\overline{\omega})\Pr(\mathrm{Y}|\overline{\omega},\, p_2) + \mu(\underline{\omega})\Pr(\mathrm{Y}|\underline{\omega},\, p_2) = \Pr(\mathrm{Y}|p_2).$$

But then I once again have a contradiction. If $s(p_1)$ is optimal at p_1 then it provides a weakly higher payoff at p_1 than $s(p_2)$ does, and, when price increases to p_2 the decrease in payoff to $s(p_1)$ is strictly smaller than the decrease in payoff to $s(p_2)$:

$$(p_2 - p_1)\Pr(\mathrm{Y}|p_1) < (p_2 - p_1)\Pr(\mathrm{Y}|p_2).$$

(iii): Again, I proceed with a proof by contradiction. Suppose $\exists p_1 \in \mathcal{P}$ such that: $\Pr(\mathrm{Y}|\underline{\omega},\, p_1) = \Pr(\mathrm{Y}|\overline{\omega},\, p_1) \in (0,\, 1)$, and $\exists p_2 \in \mathcal{P}\backslash p_1$ such that $\Pr(\mathrm{Y}|p_2) \in (0,\, 1)$. Notice that optimality of the agent's behavior at p_1 implies $s = (0,\, 0)$ and $s = (1,\, 1)$ are both optimal when price is p_1 since all information outcomes $s = (x,\, x)$ have zero cost for $x \in [0,\, 1]$. If $p_2 < p_1$, then I have a contradiction because the agent could optimally pick $s = (1,\, 1)$ at p_1, so $\Pr(\mathrm{Y}|p_1) = 1$, which combined with **(ii)** implies the agent is not behaving optimally at p_2 since then $\Pr(\mathrm{Y}|p_1) > \Pr(\mathrm{Y}|p_2)$. If $p_2 > p_1$, then I have a contradiction because the agent could optimally pick $s = (0,\, 0)$ at p_1, so $\Pr(\mathrm{Y}|p_1) = 0$, which combined with **(ii)** implies the agent is not behaving optimally at p_2 since then $\Pr(\mathrm{Y}|p_1) < \Pr(\mathrm{Y}|p_2)$.

To show **(i)**, **(ii)**, and **(iii)** are together sufficient I assume they are all satisfied and order the prices in $\mathcal{P} = \{p_1,\, p_2, \ldots,\, p_n\}$, so $p_1 < p_2 < \cdots < p_n$. Further, I can assume that there is a price in $\mathcal{P}$ where the agent learns, because if not I can easily rationalize the behavior by defining C_μ for $\underline{s} < \overline{s}$ using a steep enough plane through the diagonal of S (points $(x,\, x)$ with $x \in [0,\, 1]$). **(iii)** thus tells us that there is not behavior at a price $p_i \in \mathcal{P}$ with $s(p_i) = (x,\, x)$ and $x \in (0,\, 1)$.

It is further without loss to assume that $s(p_1) = (1,\, 1)$ and $s(p_n) = (0,\, 0)$, since if this is not the case I can add prices to $\mathcal{P}$, and data to the behavior, so that this is the case, and then rationalizing the richer dataset rationalizes the original dataset. There is then a highest $p \in \mathcal{P}$ such that $\Pr(\mathrm{Y}|p) = 1$, denote it p_h, and a lowest $p \in \mathcal{P}$ such that $\Pr(\mathrm{Y}|p) = 0$, denote it p_l. Notice $p_h < p_l$. Pick a mean $m = p_h$, which is a reference point for conducting mean preserving spreads on $u(\underline{\omega})$ and $u(\overline{\omega})$ and is shifted at the end of the argument. Then pick $u(\underline{\omega}) < p_1$ and $u(\overline{\omega}) > p_n$ such that $\mu(\underline{\omega})u(\underline{\omega}) + \mu(\overline{\omega})u(\overline{\omega}) = m$.

Remember, if there is a price $p \in \mathcal{P}$ such that $\underline{s}(p) = \overline{s}(p)$, then I know I must have $C_\mu(s(p)) = 0$. I now begin recursively assigning costs to the other information outcomes (defining values for $C_\mu(s(p))$). First, define:

$$C_\mu(s(p_{l-1})) = \mu(\underline{\omega})\Big(\underline{s}(p_{l-1})(u(\underline{\omega}) - p_l)\Big) + \mu(\overline{\omega})\Big(\overline{s}(p_{l-1})(u(\overline{\omega}) - p_l)\Big)$$

$$= \underline{s}(p_{l-1})(m - p_l) + \mu(\overline{\omega})\Big(\overline{s}(p_{l-1}) - \underline{s}(p_{l-1})\Big)(u(\overline{\omega}) - p_l).$$

This means the agent is indifferent between $s(p_{l-1})$ and $s(p_l)$ when price is p_l, and thus strictly prefers $s(p_{l-1})$ to $s(p_l)$ when price is p_{l-1} because when price decreases the payoff of $s(p_{l-1})$ strictly increases since there is a strictly positive probability of the agent selecting option Y when they choose $s(p_{l-1})$ (by construction) while the payoff of $s(p_l)$ is zero (by construction) and stays the same since probability of the agent selecting option Y when they choose $s(p_l)$ is zero (by construction). If this $C_\mu(s(p_{l-1}))$ is strictly positive I continue, and if it is not I do a mean preserving spread on $u(\underline{\omega})$ and $u(\overline{\omega})$ so it is, which works since $\overline{s}(p_{l-1}) > \underline{s}(p_{l-1})$ because of my above assumptions and **(iii)**.

Next, if $l - 2 > h$, I let:

$$C_\mu(s(p_{l-2})) = \underline{s}(p_{l-2})(m - p_{l-1}) + \mu(\overline{\omega})\Big(\overline{s}(p_{l-2}) - \underline{s}(p_{l-2})\Big)(u(\overline{\omega}) - p_{l-1})$$

$$-\underline{s}(p_{l-1})(m - p_{l-1}) - \mu(\overline{\omega})\Big(\overline{s}(p_{l-1}) - \underline{s}(p_{l-1})\Big)(u(\overline{\omega}) - p_{l-1}) + C_\mu(s(p_{l-1})).$$

This means the agent is indifferent between $s(p_{l-2})$ and $s(p_{l-1})$ when price is p_{l-1}, and thus weakly prefers $s(p_{l-2})$ to $s(p_{l-1})$ when price is p_{l-2}. If this $C_\mu(s(p_{l-2}))$ is strictly positive I continue, if it is not I do a mean preserving spread on $u(\underline{\omega})$ and $u(\overline{\omega})$ (updating $C_\mu(s(p_{l-1}))$ accordingly) so $C_\mu(s(p_{l-2}))$ is strictly positive, which works since the value of $-\mu(\overline{\omega})\Big(\overline{s}(p_{l-1}) - \underline{s}(p_{l-1})\Big)(u(\overline{\omega}) - p_{l-1}) + C_\mu(s(p_{l-1}))$ does not change, and $\overline{s}(p_{l-2}) > \underline{s}(p_{l-2})$ because of my assumptions above and **(iii)**.

I continue in this fashion until I have set $C_\mu(s(p_{h+1}))$. If I keep the mean m the same (equal to p_h), then the agent strictly prefers $s(p_{h+1})$ to $s(p_h)$ when price is p_{h+1}, since they prefer $s(p_{h+1})$ to $s(p_l)$, which they strictly prefer to $s(p_h)$. I now increase $u(\underline{\omega})$ and $u(\overline{\omega})$ by the same amount so that the mean m increases, and all $C_\mu(s(p_i)) > 0$ so that the equations I used to define costs are still satisfied, until the agent is indifferent between $s(p_{h+1})$ to $s(p_h)$ when price is p_{h+1}. As a result, the agent strictly prefers $s(p_h)$ to $s(p_{h+1})$ when price is p_h since there is a strictly higher unconditional chance the agent selects option Y when they pick $s(p_h)$ compared to $s(p_{h+1})$ by construction.

At each price p_i I assigned a cost to the information outcome so that the agent is indifferent at p_i between $s(p_i)$, and $s(p_{i-1})$. This implies, out of the set of information outcomes I observe in the behavior, the agent is selecting their strategy optimally at each price, since as price decreases the value of a strategy increase by the unconditional probability of choosing option Y, and as price decreases the unconditional probability of selecting option Y increases.

Now, I assign an arbitrarily high value to $C_\mu(0, 1)$ (so that $(0, 1)$ is strictly worse than $s(p)$ for all $p \in \mathcal{P}$), if I have not already assigned a value to it, and then define C_μ on $s \in S$ such that $\underline{s} < \overline{s}$ to be the the maximal convex function that is equal to the $C_\mu(s(p))$ I assigned for all $p \in \mathcal{P}$.

Why is this possible? Is it instead possible I assigned a $C_\mu(s(p_i))$ that was strictly above the relevant convex combination of other $C_\mu(s(p_j))$'s I assigned? This is not possible, as I can show with a quick proof by contradiction. Assume there is a set of prices $\tilde{\mathcal{P}} = \{p_m, \dots p_k\} \subseteq \mathcal{P}$, and a price $p_i \in \mathcal{P}$ such that there are positive weights α_j that sum to one such that $\alpha_m s(p_m) + \dots + \alpha_k s(p_k) = s(p_i)$, but at the same time $\alpha_m C_\mu(s(p_m)) + \dots + \alpha_k C_\mu(s(p_k)) < C_\mu(s(p_i))$. Since $\underline{s}(u(\underline{\omega}) - p)\mu(\underline{\omega}) + \overline{s}(u(\overline{\omega}) - p)\mu(\overline{\omega})$ is linear in the probabilities $\underline{s}$ and $\overline{s}$, this implies when price is p_i the agent strictly prefers randomizing over their strategies from the $\tilde{\mathcal{P}} = \{p_m, \dots p_k\}$ prices compared to selecting $s(p_i)$, but this implies there is a $p_j \in \tilde{\mathcal{P}} = \{p_m, \dots p_k\}$ such that the agent strictly prefers $s(p_j)$ to $s(p_i)$ at p_i, which was ruled out by my recursive definition for the costs of information outcomes.

Similarly, at each p_i the information outcome of the agent is optimal given choice from the entire set of S since at each s with $\underline{s} \leq \overline{s}$ the cost is a convex combination of costs from information outcomes used for prices in $\mathcal{P}$ and the corner $(0, 1)$, and if the agent would prefer to switch to a different s at p_i, then they strictly prefer randomizing over the set of information outcomes used to generate the cost of s, and again one of the information outcomes used to generate the cost at s would then have been strictly preferred at p_i, and my recursive definition for the cost of information outcomes (and choice of arbitrarily high $C_\mu(0, 1)$) rules this out. ■

Lemma 1. Suppose I am given a price p, a prior belief μ, and an ex-ante cost function for information outcomes C_μ, such that:

$$s(p) = (\underline{s}(p),\, \overline{s}(p)) \in \underset{(\underline{s},\, \overline{s}) \in S}{\arg\max} \Big(\underline{s}\mu(\underline{\omega})(u(\underline{\omega}) - p) + \overline{s}\mu(\overline{\omega})(u(\overline{\omega}) - p) - C_\mu(\underline{s},\, \overline{s}) \Big),$$

and $\underline{s}(p) < \overline{s}(p)$. Then I can always find a $q \in (\frac{1}{2},\, 1]$ such that $s = (1-q,\, q)$ is an optimal information outcome in the following way:

(i) If $\Pr(\mathrm{Y}|\overline{\omega},\, p) = \Pr(\mathrm{X}|\underline{\omega},\, p) = q$, then $s = (1-q,\, q)$ is already optimal.

(ii) If $\Pr(\mathrm{Y}|\overline{\omega},\, p) > \Pr(\mathrm{X}|\underline{\omega},\, p)$, then if I draw a line through $s = (1,\, 1)$ and $s(p)$ then the point where this line hits the line defined by $s = (1-q,\, q)$ is optimal, which is to say there is a unique $q \in (\frac{1}{2},\, 1]$, and a unique $\alpha \in [0,\, 1]$, such that $\alpha(1-q) + (1-\alpha)1 = \Pr(Y|\underline{\omega},\, p)$, and $\alpha q + (1-\alpha)1 = \Pr(Y|\overline{\omega},\, p)$, and further, $s = (1-q,\, q)$ is optimal:

$$(1-q,\, q) \in \underset{(\underline{s},\, \overline{s}) \in S}{\arg\max} \Big(\underline{s}\mu(\underline{\omega})(u(\underline{\omega}) - p) + \overline{s}\mu(\overline{\omega})(u(\overline{\omega}) - p) - C_\mu(\underline{s},\, \overline{s}) \Big).$$

(iii) If $\Pr(\mathrm{Y}|\overline{\omega},\, p) < \Pr(\mathrm{X}|\underline{\omega},\, p)$, then if I draw a line through $s = (0,\, 0)$ and $s(p)$ then the point where this line hits the line defined by $s = (1-q,\, q)$ is optimal, which is to say there is a unique $q \in (\frac{1}{2},\, 1]$, and a unique $\alpha \in [0,\, 1]$, such that $\alpha(1-q) + (1-\alpha)0 = \Pr(Y|\underline{\omega},\, p)$, and $\alpha q + (1-\alpha)0 = \Pr(Y|\overline{\omega},\, p)$, and further, $s = (1-q,\, q)$ is optimal:

$$(1-q,\, q) \in \underset{(\underline{s},\, \overline{s}) \in S}{\arg\max} \Big(\underline{s}\mu(\underline{\omega})(u(\underline{\omega}) - p) + \overline{s}\mu(\overline{\omega})(u(\overline{\omega}) - p) - C_\mu(\underline{s},\, \overline{s}) \Big).$$

Proof The first case is trivial. For the second case, consider the q defined by the equation. If the agent learns and $\Pr(\mathrm{Y}|\overline{\omega},\, p) > \Pr(\mathrm{X}|\underline{\omega},\, p)$, then given how I defined ex-ante cost functions, there must be an optimal information outcome on the red line ($(x,\, x)$ line) from Figure 2 and an optimal information outcome on the the blue line ($(1-q,\, q)$ line) from Figure 2 whose convex combination is $s(p)$, and there must be a $\tilde{q} \geq q$ that is optimal (if I move away from $(1,\, 1)$ on the red line my rotation gives us higher q). If the point on the red line is not $(1,\, 1)$, then I can infer it is optimal for the agent to select $(\frac{1}{2},\, \frac{1}{2})$, and if there is $\tilde{q} \geq q$ that is also optimal, then since optimal sets need to be convex, q is optimal.

For the third case, consider the q defined by the equation. If the agent learns and $\Pr(\mathrm{Y}|\overline{\omega},\, p) < \Pr(\mathrm{X}|\underline{\omega},\, p)$, then given how I defined ex-ante cost functions, there must be an optimal information outcome on the red line ($(x,\, x)$ line) from Figure 2 and an optimal information outcome on the

the blue line ($(1-q,\, q)$ line) from Figure 2 whose convex combination is $s(p)$, and there must be a $\tilde{q} \geq q$ that is optimal (if I move away from $(0,\, 0)$ on the red line my rotation gives us higher q). If the point on the red line is not $(0,\, 0)$, then I can infer it is optimal for the agent to select $(\frac{1}{2},\, \frac{1}{2})$, and then since optimal sets need to be convex, q is optimal. ■

Proof of Proposition 2. **(i)** is necessary because if there are two prices $p_1 < p_2$ such that $\Pr(\mathrm{Y}|\overline{\omega},\, p) > \Pr(\mathrm{X}|\underline{\omega},\, p) > 0$, then at each price $(1,\, 1)$ is optimal (because of construction of ex-ante accuracy cost functions), and the agent could have selected it at p_2, but then Proposition 1 tells us they were not selecting an optimal strategy at p_1. **(ii)** is necessary because if there are two prices $p_1 < p_2$ such that $\Pr(\mathrm{X}|\overline{\omega},\, p) > \Pr(\mathrm{Y}|\underline{\omega},\, p) > 0$, then at each price $(0,\, 0)$ is optimal (because of construction of ex-ante accuracy cost functions), and the agent could have selected it at p_1, but then Proposition 1 tells us they were not selecting an optimal strategy at p_2. **(iii)** is necessary because if not, I also get a violation of Proposition 1 since the respective q's were both optimal, the agent could have selected information outcomes for the two prices such that $s = (1-q,\, q)$ for the two q's, and then the unconditional chance of selecting Y given either q is $\mu(\overline{\omega})q + \mu(\underline{\omega})(1-q)$, which means the unconditional chance of selecting Y increased when price increased, contradicting Proposition 1.

Sufficiency is easy given the result in Lemma 1 and the recursive definition for the costs of information outcomes from the proof of Proposition 1. Order the prices in $\mathcal{P} = \{p_1,\, p_2, \ldots,\, p_n\}$, so $p_1 < p_2 < \cdots < p_n$. Further, I can assume that there is a price in $\mathcal{P}$ where the agent learns, because if not I can easily rationalize the behavior by defining a C_μ for $\underline{s} < \overline{s}$ using a steep enough plane through the diagonal of S (points $(x,\, x)$ with $x \in [0,\, 1]$). **(iii)** thus tells us that there is not behavior at a price $p_i \in \mathcal{P}$ with $s(p_i) = (x,\, x)$ and $x \in (0,\, 1)$. It is further without loss to assume that $s(p_1) = (1,\, 1)$ and $s(p_n) = (0,\, 0)$, since if this is not the case I can add prices to $\mathcal{P}$, and data to the behavior, so that this is the case, and then rationalizing the richer dataset rationalizes the original dataset.

It is further without loss to assume that if the agent learns at a price p_i, then $s(p_i)$ is such that $1 - \underline{s}(p_i) = \overline{s}(p_i)$. If not, and $1 - \underline{s}(p_i) > \overline{s}(p)$, then change $s(p_i) = (0, 0)$ and enrich the behavior so there is a $p_j \in \mathcal{P}$ so $p_j \in (p_{i-1},\, p_i)$ and $s(p_j) = (1-q,\, q)$ where q is the optimal one at p_i according to Lemma 1. If not, and $1 - \underline{s}(p_i) < \overline{s}(p_i)$, then change $s(p_i) = (1-q,\, q)$ where q is the optimal one at p_i according to Lemma 1. Then, I can just use the same recursive definition for the costs of information outcomes as in the proof of Proposition 1 to rationalize the behavior (rationalizes the behavior before and after my without loss change), and the resultant C_μ is an ex-ante cost function for information outcomes (see proof of Proposition 1). ■

Proof of Proposition 4. I assume there is a $p \in \mathcal{P}$ such that $\underline{s}(p) < \overline{s}(p)$, otherwise the necessary conditions are trivially established, and sufficiency is easy to establish by making learning costly enough (since belief is fixed).

When the agent pays for information according to a posterior separable cost function (weakly convex c), the way for them to maximize their expected payoff is to maximize the weighted average over option specific net utilities $V(\mathrm{Y}|p, \cdot)$ and $V(\mathrm{X}|\cdot)$, defined in the next paragraph. Each option specific net utility takes into account the expected payoff of the relevant option, X or Y, given the agent's posterior, and the cost of the posterior reached by the agent when they choose said option, where the posterior can be described by the resultant probability of $\overline{\omega}$ being realized, $\Pr(\overline{\omega}|\mathrm{Y}, p)$ or $\Pr(\overline{\omega}|\mathrm{X}, p)$ respectively.

If the agent does no learning they choose X or Y depending on the expected value of selecting option Y, $m - p = \mu(\underline{\omega})u(\underline{\omega}) + \mu(\overline{\omega})u(\overline{\omega}) - p$, and which is larger as a result, the value selecting option Y at the prior, $V(\mathrm{Y}|p, \mu(\overline{\omega})) = m - p$, or X at the prior, $V(\mathrm{X}|\mu(\overline{\omega})) = 0$. If the agent does some learning, then when they choose X their payoff is $V(\mathrm{X}|\Pr(\overline{\omega}|\mathrm{X}, p)) = -c(\Pr(\overline{\omega}|\mathrm{X}, p)) + c(\mu(\overline{\omega}))$, and when they select Y their payoff is $V(\mathrm{Y}|p, \Pr(\overline{\omega}|\mathrm{Y}, p)) = \Pr(\overline{\omega}|\mathrm{Y}, p)(u(\overline{\omega}) - p) + (1 - \Pr(\overline{\omega}|\mathrm{Y}, p))(u(\underline{\omega}) - p) - c(\Pr(\overline{\omega}|\mathrm{Y}, p)) + c(\mu(\overline{\omega}))$.

Both $V(\mathrm{Y}|p, \cdot)$ and $V(\mathrm{X}|\cdot)$ are weakly concave functions since c is weakly convex. The agent is maximizing the weighted average of the two. Notice that in any optimal solution, if the agent is learning, $\Pr(\overline{\omega}|\mathrm{X}, p) < \mu(\overline{\omega}) < \Pr(\overline{\omega}|\mathrm{Y}, p)$. As a result, to find the optimal solution of the agent I must find the cord from the top of $V(\mathrm{X}|\cdot)$ on the left of $\mu(\overline{\omega})$ to the top of $V(\mathrm{Y}|p, \cdot)$ on the right, so that I have a weakly concave closure.

When p increases, either the agent stops learning, which means the concave closure at $\mu(\overline{\omega})$ is $V(\mathrm{X}|\mu(\overline{\omega}))$, or the agent continues to learn, in which case, $V(\mathrm{Y}|p, \cdot)$ shifts downward at every point by the change in price, and the point where the cord that creates the weakly concave closure hits $V(\mathrm{Y}|p, \cdot)$ and $V(\mathrm{X}|\cdot)$ must then both weakly move to the right, which means $\Pr(\overline{\omega}|\mathrm{X}, p)$ and $\Pr(\overline{\omega}|\mathrm{Y}, p)$ are both weakly increasing, which establishes necessity.

Next I show sufficiency by constructing a weakly convex c function that generates the observed behavior. It is without loss to assume $\mathcal{P} = \{p_1, p_2, \ldots, p_n\}$, with $p_1 < \cdots < p_n$, and $\Pr(\mathrm{Y}|\Omega, \mathcal{P})$ are such that $n \geq 4$ (in the example graphs, $n = 6$), with $\Pr(\mathrm{Y}|p_1) = 1$, $\Pr(\mathrm{Y}|p_n) = 0$, and

$\Pr(\mathrm{Y}|p_i) \in (0,\, 1)$ for $p_i \in \mathcal{P}\backslash\{p_1,\, p_n\}$, since if this is not the case I can generate such a dataset by adding more prices with behavior that satisfy the conditions since rationalizing this richer dataset rationalizes the original dataset.

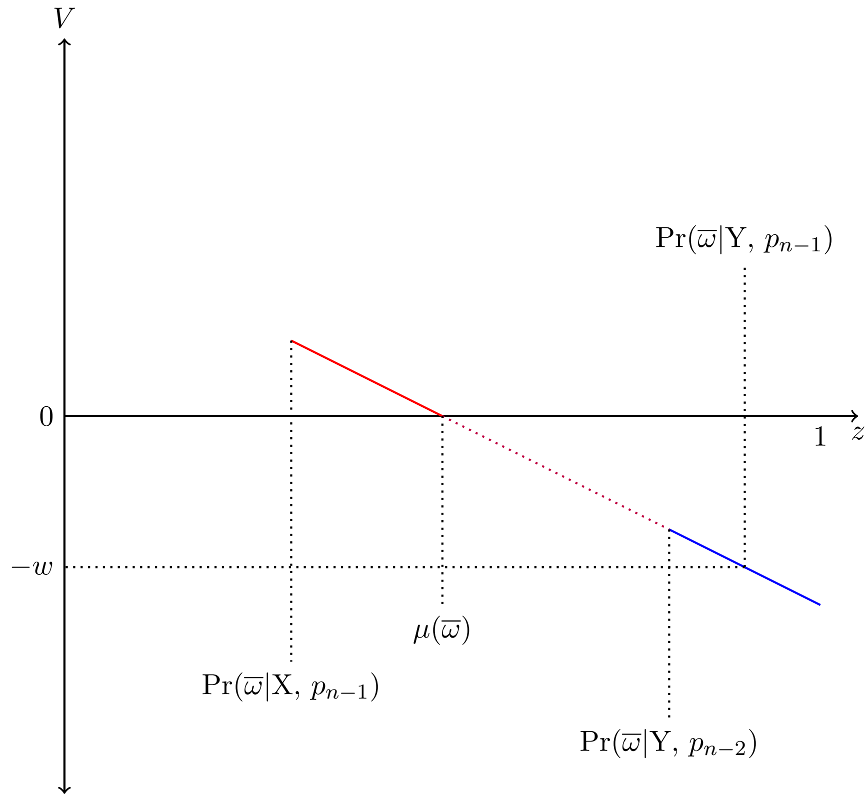
I am now going to draw a graph step by step. After I draw the graph, the different components are going to help us pick $u(\underline{\omega})$ and $u(\overline{\omega})$, and tell us a suitable c. The horizontal axis goes from zero to one, I call the horizontal coordinate z since X is already being used. The vertical axis may take positive and negative values, I call this coordinate height, whether it be positive or negative, since Y is being used.

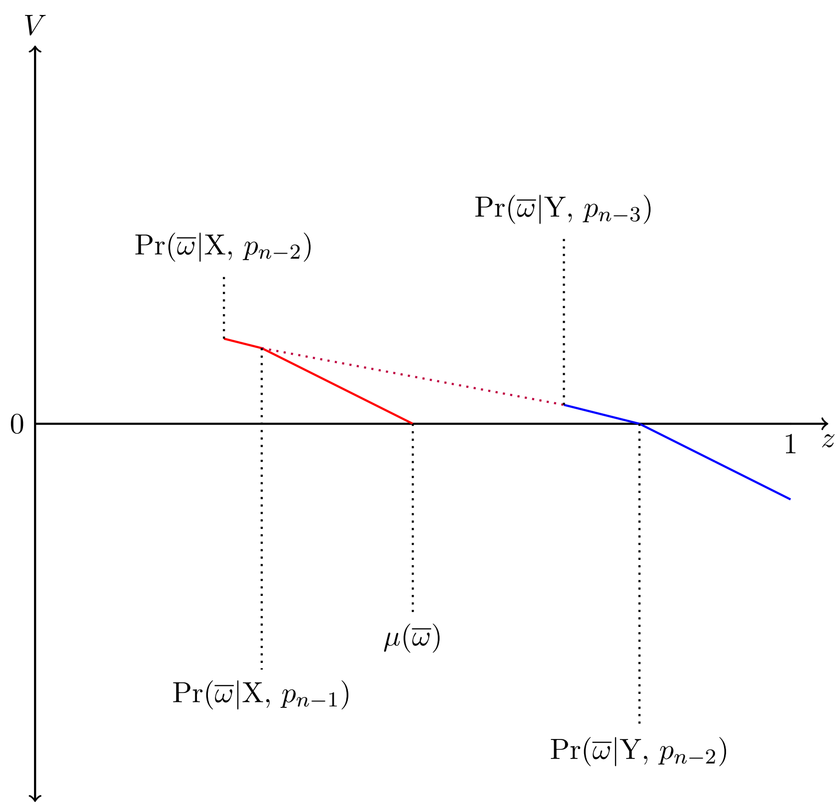
First (first graph): I draw a line segment from $z = \Pr(\overline{\omega}|\mathrm{X},\, p_{n-1})$ and a positive height, to $z = 1$ and a negative height, so that when $z = \mu(\overline{\omega})$, the height is 0, and so that the height at $\Pr(\overline{\omega}|\mathrm{Y},\, p_{n-1})$ is $-w$ such that $p_{n-1}-p_2-w > 0$. The part of the segment between $z = \Pr(\overline{\omega}|\mathrm{X},\, p_{n-1})$ and a $z = \mu(\overline{\omega})$ I make red, and the segment between $z = \Pr(\overline{\omega}|\mathrm{Y},\, p_{n-2})$ and $z = 1$ I make blue, the rest of this first line segment that I did not make blue or red I can erase (in graph as purple dots).

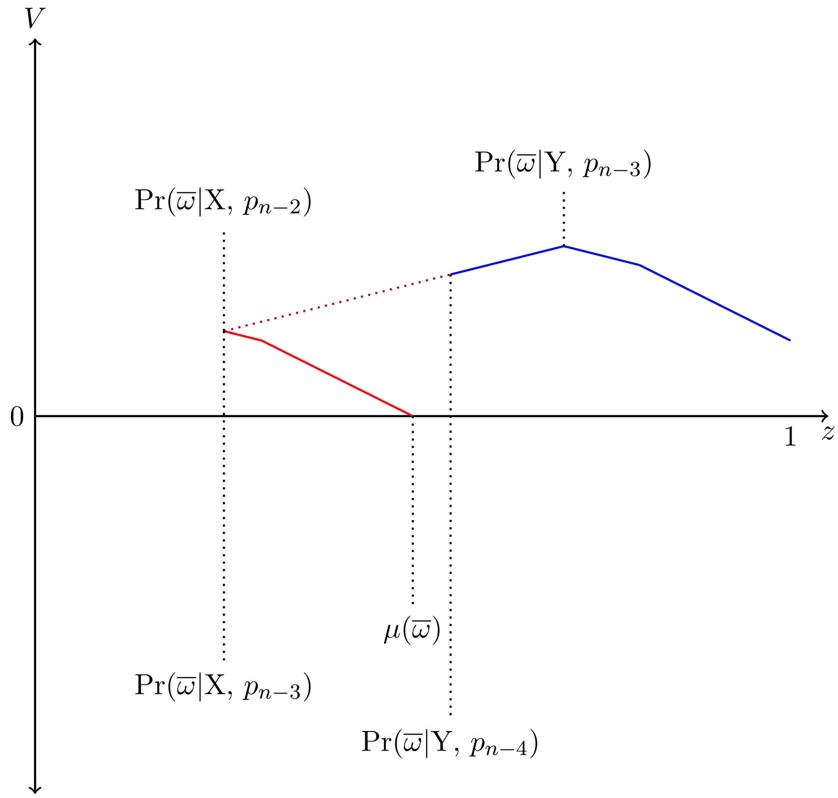
Second (second graph): I take the blue segment, and at each z I increase the height by $p_{n-1} - p_{n-2}$, then I draw a new line segment from the blue line at $z = \Pr(\overline{\omega}|\mathrm{Y},\, p_{n-2})$, through the red segment at $z = \Pr(\overline{\omega}|\mathrm{X},\, p_{n-1})$ and continue on to $z = \Pr(\overline{\omega}|\mathrm{X},\, p_{n-2})$. The part of the new segment between $z = \Pr(\overline{\omega}|\mathrm{X},\, p_{n-1})$ and $z = \Pr(\overline{\omega}|\mathrm{X},\, p_{n-2})$ I make red, and the part of the new segment between $z = \Pr(\overline{\omega}|\mathrm{Y},\, p_{n-2})$ and $z = \Pr(\overline{\omega}|\mathrm{Y},\, p_{n-3})$ I make blue, the rest of the new line segment that I did not make blue or red I can erase (in graph as purple dots).

Third (third graph): I take the blue segment, and at each z I increase the height by $p_{n-2} - p_{n-3}$, then I draw a new line segment from the blue line at $z = \Pr(\overline{\omega}|\mathrm{Y},\, p_{n-3})$, through the red segment at $z = \Pr(\overline{\omega}|\mathrm{X},\, p_{n-2})$ and continue on to $z = \Pr(\overline{\omega}|\mathrm{X},\, p_{n-3})$. The part of the new segment between $z = \Pr(\overline{\omega}|\mathrm{X},\, p_{n-2})$ and $z = \Pr(\overline{\omega}|\mathrm{X},\, p_{n-3})$ I make red, and the part of the new segment between $z = \Pr(\overline{\omega}|\mathrm{Y},\, p_{n-3})$ and $z = \Pr(\overline{\omega}|\mathrm{Y},\, p_{n-4})$ I make blue, the rest of the new line segment that I did not make blue or red I can erase (in graph as purple dots). In the example graphs, $\Pr(\overline{\omega}|\mathrm{X},\, p_{n-2}) = \Pr(\overline{\omega}|\mathrm{X},\, p_{n-3})$, which is meant to provide some insight into what happens when $\Pr(\overline{\omega}|\mathrm{X},\, p)$ or $\Pr(\overline{\omega}|\mathrm{Y},\, p)$ are only weakly decreasing in p.

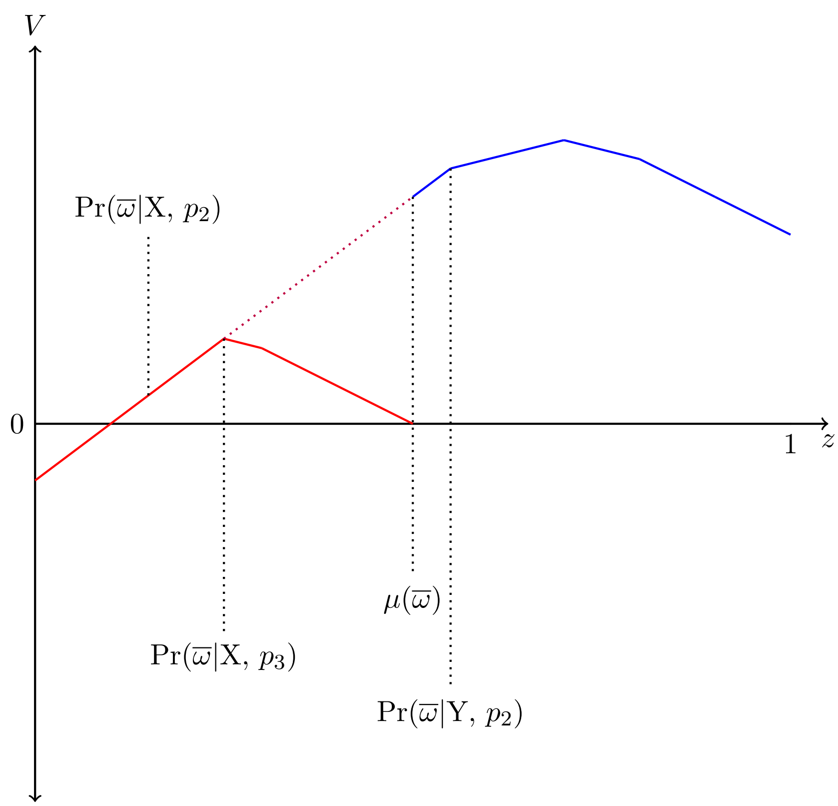
Eventually (fourth graph), after continuing in the above fashion, I take the blue segment, and at each z I increase the height by $p_3 - p_2$, then I draw a new line segment from the blue line at $z = \Pr(\overline{\omega}|\mathrm{Y},\, p_2)$, through the red segment at $z = \Pr(\overline{\omega}|\mathrm{X},\, p_3)$ and continue on to $z = 0$. The part of the new segment between $z = \Pr(\overline{\omega}|\mathrm{X},\, p_3)$ and $z = 0$ I make red, and the part of the new


V
$\Pr(\overline{\omega}|\mathrm{Y},\, p_{n-1})$
0
1
z
$-w$
$\mu(\overline{\omega})$
$\Pr(\overline{\omega}|\mathrm{X},\, p_{n-1})$
$\Pr(\overline{\omega}|\mathrm{Y},\, p_{n-2})$


V
$\Pr(\overline{\omega}|\mathrm{Y},\, p_{n-3})$
$\Pr(\overline{\omega}|\mathrm{X},\, p_{n-2})$
0
1
z
$\mu(\overline{\omega})$
$\Pr(\overline{\omega}|\mathrm{X},\, p_{n-1})$
$\Pr(\overline{\omega}|\mathrm{Y},\, p_{n-2})$



V
$\Pr(\overline{\omega}|\mathrm{Y},\, p_{n-3})$
$\Pr(\overline{\omega}|\mathrm{X},\, p_{n-2})$
0
1
z
$\mu(\overline{\omega})$
$\Pr(\overline{\omega}|\mathrm{X},\, p_{n-3})$
$\Pr(\overline{\omega}|\mathrm{Y},\, p_{n-4})$



V
$\Pr(\overline{\omega}|\mathrm{X},\, p_2)$
0
1
z
$\mu(\overline{\omega})$
$\Pr(\overline{\omega}|\mathrm{X},\, p_3)$
$\Pr(\overline{\omega}|\mathrm{Y},\, p_2)$

segment between $z = \mu(\overline{\omega})$ and $z = \Pr(\overline{\omega}|\mathrm{Y},\, p_2)$ I make blue, the rest of the new line segment that I did not make blue or red I can erase (in graph as purple dots).

The red line segments I take to be $-c(z) + c(\mu(\overline{\omega}))$ for $z \in [0,\, \mu(\overline{\omega})]$. Next, let $b(z)$ denote the height of the blue segment for $z \in [\mu(\overline{\omega}),\, 1]$ (in the final graph). $b(\mu(\overline{\omega})) > 0$ because either the slope of the blue segments is negative (where it is defined), in which case $b(\mu(\overline{\omega}))$ is more than $b(\Pr(\overline{\omega}|\mathrm{Y},\, p_{n-1}))$, which is strictly positive based on how I chose $-w$ at the beginning, or the slope of the blue segments is positive or zero immediately to the right of $z = \mu(\overline{\omega})$, in which case $b(\mu(\overline{\omega}))$ is greater or equal to the height of the red segments at $\Pr(\overline{\omega}|\mathrm{X},\, p_{n-1})$, which is strictly positive by construction. Further, $b(\mu(\overline{\omega})) - p_{n-1} + p_2 < 0$, again based on how I picked w. Next I pick the mean quality to be $m = p_2 + b(\mu(\overline{\omega})) \in (p_2,\, p_{n-1})$ so that when $p = m$ the agent is indifferent between choosing X without learning and choosing Y without learning. Next, I pick $u(\underline{\omega})$ and $u(\overline{\omega})$ so that $\mu(\underline{\omega})u(\underline{\omega}) + \mu(\overline{\omega})u(\overline{\omega}) = m$, and so that:

$$u(\overline{\omega}) - u(\underline{\omega}) + \frac{c(\mu(\overline{\omega})) - c(\Pr(\overline{\omega}|\mathrm{X},\, p_{n-1})}{\mu(\overline{\omega}) - \Pr(\overline{\omega}|\mathrm{X},\, p_{n-1})} \geq \frac{b(\Pr(\overline{\omega}|\mathrm{Y},\, p_2)) - b(\mu(\overline{\omega}))}{\Pr(\overline{\omega}|\mathrm{Y},\, p_2) - \mu(\overline{\omega})}.$$

Next, for $z \in [\mu(\overline{\omega}),\, 1]$ I let $-c(z) + c(\mu(\overline{\omega})) = b(z) - b(\mu(\overline{\omega})) - (z - \mu(\overline{\omega}))(u(\overline{\omega}) - u(\underline{\omega}))$. Finally, I fix $c(\mu(\overline{\omega}))$ so that $\min_{z \in [0,\, 1]} c(z) = 0$. ■

Proof of Corollary 2. If the behavior is rationalized by an exogenous learning model, then it is rationalized by a costly learning model. But, suppose it is not rationalized by an ex-post accuracy model. Then there are two prices $p_1 < p_2$, both in $\mathcal{P}$ at which the agent learns such that either $\Pr(\overline{\omega}|\mathrm{X},\, p_1) > \Pr(\overline{\omega}|\mathrm{X},\, p_2)$, or $\Pr(\overline{\omega}|\mathrm{Y},\, p_1) > \Pr(\overline{\omega}|\mathrm{Y},\, p_2)$. Let $\underline{\delta} = \Pr(\mathrm{Y}|\underline{\omega},\, p_1) - \Pr(\mathrm{Y}|\underline{\omega},\, p_2)$, and let $\overline{\delta} = \Pr(\mathrm{Y}|\overline{\omega},\, p_1) - \Pr(\mathrm{Y}|\overline{\omega},\, p_2)$. If $\overline{\delta} < 0$, then Proposition 1 requires $\underline{\delta} > 0$, but this is not possible since all information outcome costs are zero, then the agent was not choosing an optimal information outcome at p_1. If $\underline{\delta} < 0$, then Proposition 1 requires $\overline{\delta} > 0$, but this is not possible since all information outcome costs are zero, then the agent was not choosing an optimal information outcome at p_2. So, exogenous learning implies $\overline{\delta},\, \underline{\delta} \geq 0$. So, the change in behavior is characterized by a lower chance of selecting Y given either value when price increases. The expected value of Y in the region where behavior changed is:

$$\frac{\underline{\delta}\mu(\underline{\omega})}{\underline{\delta}\mu(\underline{\omega}) + \overline{\delta}\mu(\overline{\omega})} u(\underline{\omega}) + \frac{\overline{\delta}\mu(\overline{\omega})}{\underline{\delta}\mu(\underline{\omega}) + \overline{\delta}\mu(\overline{\omega})} u(\overline{\omega}).$$

If $\Pr(\overline{\omega}|\mathrm{X},\, p_1) > \Pr(\overline{\omega}|\mathrm{X},\, p_2)$, then the expected value of Y in the region where behavior changed is strictly lower than the expected value of Y after a the agent chose X at p_1, which must be weakly less than p_1, so the agent was not choosing their information outcome optimally at p_1 (remember all chosen information outcomes are costless). Similarly, if $\Pr(\overline{\omega}|\text{Y}, p_1) > \Pr(\overline{\omega}|\text{Y}, p_2)$, then the expected value of Y in the region where behavior changed is strictly higher than the expected value of Y after the agent chose Y at p_2, which must be weakly more than p_2, so the agent was not choosing their information outcome optimally at p_2 (remember all chosen information outcomes are costless). ■

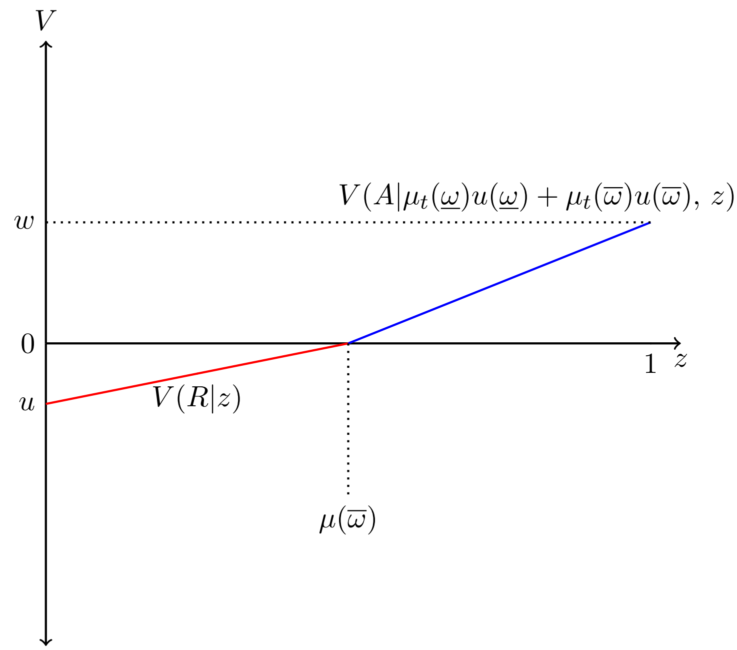
Proof of Proposition 5. I begin with a helpful lemma.

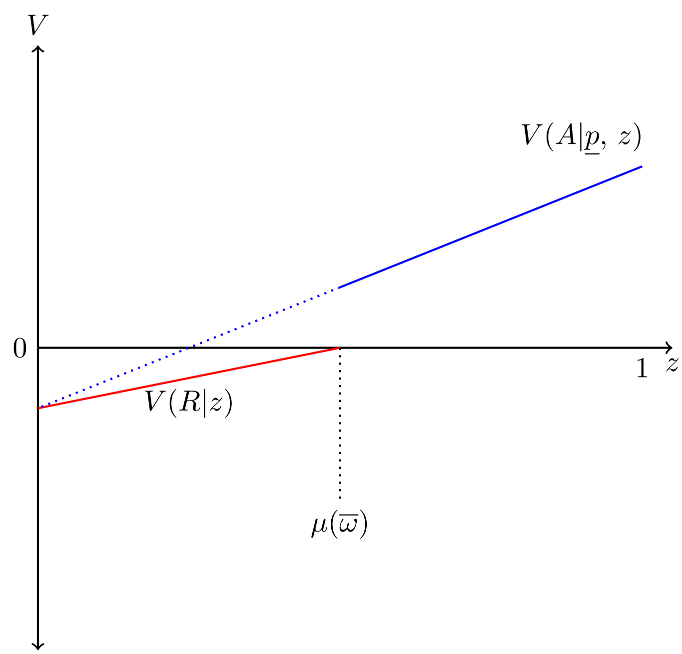
Lemma 2. Given prices $\underline{p}$ and $\overline{p} > \underline{p}$, values $u(\underline{\omega})$ and $u(\overline{\omega})$ such that $u(\underline{\omega}) \leq \underline{p}$ and $u(\overline{\omega}) \geq \overline{p}$, and a type t with a prior belief such that $\mu_t(\overline{\omega}) \in (0, 1)$ and $\mu_t(\underline{\omega}) \equiv 1 - \mu_t(\overline{\omega})$, I can create a posterior separable cost function $c_t : [0, 1] \to \mathbb{R}_+$ such that agents of type t are indifferent between doing no learning ($\underline{s} = \overline{s}$) and perfectly observing the value of option Y ($\underline{s} = 0$, $\overline{s} = 1$) when the price is $p \in \{\underline{p}, \overline{p}\}$, strictly prefers perfectly observing the value of option Y over all other learning strategies when the price is $p \in (\underline{p}, \overline{p})$, and strictly prefers doing no learning over all other learning strategies when $p \notin [\underline{p}, \overline{p}]$, if the values are a mean preserving spread of the prices:

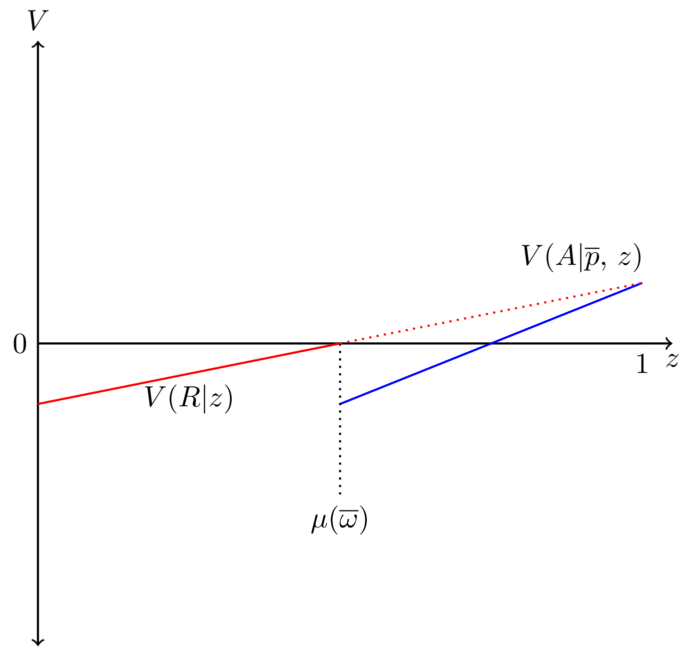
$$\mu_t(\underline{\omega})u(\underline{\omega}) + \mu_t(\overline{\omega})u(\overline{\omega}) = \mu_t(\underline{\omega})\underline{p} + \mu_t(\overline{\omega})\overline{p}.$$

Proof of Lemma 2. In the fashion of the proof of Proposition 4, I construct the functions $V(R|z)$ in red on the left of $\mu_t(\overline{\omega})$ and $V(A|p, z)$ in blue on the right of $\mu_t(\overline{\omega})$ (graphs below).

When $p = \mu_t(\underline{\omega})u(\underline{\omega}) + \mu_t(\overline{\omega})u(\overline{\omega})$, these functions must both be equal to zero. I am going to draw $V(R|z)$ and $V(A|\mu_t(\underline{\omega})u(\underline{\omega}) + \mu_t(\overline{\omega})u(\overline{\omega}), z)$ as line segments on either side of $\mu_t(\overline{\omega})$, so that $V(R|\mu_t(\overline{\omega})) = V(A|\mu_t(\underline{\omega})u(\underline{\omega}) + \mu_t(\overline{\omega})u(\overline{\omega}), \mu_t(\overline{\omega})) = 0$, and so that they satisfy two other properties. **(i)**: First, when the blue line segment is shifted up by $\mu_t(\underline{\omega})u(\underline{\omega}) + \mu_t(\overline{\omega})u(\overline{\omega}) - \underline{p}$, it must be that if I were to extend the blue line segment so that it reaches $z = 0$, it hits the red line segment at $z = 0$, so that when the $p = \underline{p}$ the agent is indifferent between learning nothing and everything, and at all lower prices they strictly prefer learning nothing. **(ii)**: Second, it must be that when the blue line segment is shifted down by $\overline{p} - \mu_t(\underline{\omega})u(\underline{\omega}) + \mu_t(\overline{\omega})u(\overline{\omega})$, it must be that if I were to extend the red line segment so that it reaches $z = 1$, it hits the blue line segment at $z = 1$, so that when $p = \overline{p}$ the agent is indifferent between learning nothing and everything, and at all higher prices the agent strictly prefers learning nothing. Together, these properties imply slope of the blue line segment is strictly greater than the slope of the red line segment, and the agent strictly prefers learning everything over all other learning strategies when $p \in (\underline{p}, \overline{p})$.







How do I find line segments that satisfy the two properties? I select the height of the red segment at zero (t in the first graph), and the blue segment at one (w in the first graph), so that:

$$\textbf{(i)}:\ \frac{w}{\mu_t(\underline{\omega})} = b+w+(\mu_t(\underline{\omega})u(\underline{\omega})+\mu_t(\overline{\omega})u(\overline{\omega})-\underline{p}),\ \textbf{(ii)}:\ \frac{b}{\mu_t(\overline{\omega})} = b+w-(\overline{p}-\mu_t(\underline{\omega})u(\underline{\omega})-\mu_t(\overline{\omega})u(\overline{\omega}))$$

$$\Leftrightarrow$$

$$\frac{\mu_t(\overline{\omega})w}{\mu_t(\underline{\omega})} = b + (\mu_t(\underline{\omega})u(\underline{\omega}) + \mu_t(\overline{\omega})u(\overline{\omega}) - \underline{p}),\ \frac{\mu_t(\underline{\omega})b}{\mu_t(\overline{\omega})} = w - (\overline{p} - \mu_t(\underline{\omega})u(\underline{\omega}) - \mu_t(\overline{\omega})u(\overline{\omega}))$$

$$\Leftrightarrow$$

$$b = \frac{\mu_t(\overline{\omega})w}{\mu_t(\underline{\omega})} - (\mu_t(\underline{\omega})u(\underline{\omega}) + \mu_t(\overline{\omega})u(\overline{\omega}) - \underline{p}),\ b = \frac{\mu_t(\overline{\omega})w}{\mu_t(\underline{\omega})} - \frac{\mu_t(\overline{\omega})}{\mu_t(\underline{\omega})}(\overline{p} - \mu_t(\underline{\omega})u(\underline{\omega}) - \mu_t(\overline{\omega})u(\overline{\omega})).$$

But, since:

$$\mu_t(\underline{\omega})u(\underline{\omega})+\mu_t(\overline{\omega})u(\overline{\omega}) = \mu_t(\underline{\omega})\underline{p}+\mu_t(\overline{\omega})\overline{p} \Rightarrow -(\mu_t(\underline{\omega})u(\underline{\omega})+\mu_t(\overline{\omega})u(\overline{\omega})-\underline{p}) = -\frac{\mu_t(\overline{\omega})}{\mu_t(\underline{\omega})}(\overline{p}-\mu_t(\underline{\omega})u(\underline{\omega})-\mu_t(\overline{\omega})u(\overline{\omega})),$$

$$b = \frac{\mu_t(\overline{\omega})w}{\mu_t(\underline{\omega})} - (\mu_t(\underline{\omega})u(\underline{\omega}) + \mu_t(\overline{\omega})u(\overline{\omega}) - \underline{p}) \Leftrightarrow b = \frac{\mu_t(\overline{\omega})w}{\mu_t(\underline{\omega})} - \frac{\mu_t(\overline{\omega})}{\mu_t(\underline{\omega})}(\overline{p} - \mu_t(\underline{\omega})u(\underline{\omega}) - \mu_t(\overline{\omega})u(\overline{\omega})).$$

Thus, given any b, I simply make $w = \frac{\mu_t(\underline{\omega})}{\mu_t(\overline{\omega})}b + (\overline{p} - \mu_t(\underline{\omega})u(\underline{\omega}) - \mu_t(\overline{\omega})u(\overline{\omega}))$.

For $z \in [0,\, \mu(\overline{\omega})]$, I let $-c(z) + c(\mu(\overline{\omega})) = V(R|z)$. For $z \in [\mu(\overline{\omega}),\, 1]$, I let $-c(z) + c(\mu(\overline{\omega})) = V(A|\mu_t(\underline{\omega})u(\underline{\omega}) + \mu_t(\overline{\omega})u(\overline{\omega}),\, z) - (z - \mu(\overline{\omega}))(u(\overline{\omega}) - u(\underline{\omega}))$. Finally, fix $c(\mu(\overline{\omega}))$ so that

$\min_{z\in[0,1]} c(z) = 0$. ■

I now return to the **proof of Proposition 5.** I begin by showing that for any type t it must be that $\Pr_t(\mathrm{Y}|\underline{\omega}, p)$ and $\Pr_t(\mathrm{Y}|\overline{\omega}, p)$ are weakly decreasing in p. If $\Pr_t(\mathrm{Y}|\underline{\omega}, p) = \Pr_t(\mathrm{Y}|\overline{\omega}, p) = 1$ clearly both need to decrease. If $\Pr_t(\mathrm{Y}|\underline{\omega}, p) < \Pr_t(\mathrm{Y}|\overline{\omega}, p) \leq 1$, then if $\Pr_t(\mathrm{Y}|\underline{\omega}, p)$ increases, Proposition 1 requires $\Pr_t(\mathrm{Y}|\overline{\omega}, p)$ decreases, and Bayes' Rule tells us:

$$\Pr_t(\overline{\omega}|\mathrm{Y}, p) = \frac{\Pr_t(\mathrm{Y}|\overline{\omega}, p)\mu_t(\overline{\omega})}{\Pr_t(\mathrm{Y}|\underline{\omega}, p)\mu_t(\underline{\omega}) + \Pr_t(\mathrm{Y}|\overline{\omega}, p)\mu_t(\overline{\omega})} = \frac{1}{1 + \dfrac{\Pr_t(\mathrm{Y}|\underline{\omega}, p)\mu_t(\underline{\omega})}{\Pr_t(\mathrm{Y}|\overline{\omega}, p)\mu_t(\overline{\omega})}},$$

so $\Pr_t(\overline{\omega}|\mathrm{Y}, p)$ decreases, which violates Proposition 4. If $\Pr_t(\mathrm{Y}|\underline{\omega}, p) \leq \Pr_t(\mathrm{Y}|\overline{\omega}, p) < 1$, and $\Pr_t(\mathrm{Y}|\overline{\omega}, p)$ increases, then Proposition 1 tells us $\Pr_t(\mathrm{Y}|\underline{\omega}, p)$ decreases, and Bayes' Rule tells us:

$$\Pr_t(\overline{\omega}|\mathrm{X}, p) = \frac{\Pr_t(\mathrm{X}|\overline{\omega}, p)\mu_t(\overline{\omega})}{\Pr_t(\mathrm{X}|\underline{\omega}, p)\mu_t(\underline{\omega}) + \Pr_t(\mathrm{X}|\overline{\omega}, p)\mu_t(\overline{\omega})} = \frac{1}{1 + \dfrac{\Pr_t(\mathrm{X}|\underline{\omega}, p)\mu_t(\underline{\omega})}{\Pr_t(\mathrm{X}|\overline{\omega}, p)\mu_t(\overline{\omega})}},$$

so $\Pr_t(\overline{\omega}|\mathrm{X}, p)$ decreases, which violates Proposition 4. As a result $\Pr(\mathrm{Y}|\underline{\omega}, p)$ and $\Pr(\mathrm{Y}|\overline{\omega}, p)$ both being weakly decreasing is necessary for $\Pr(\mathrm{Y}|\Omega, \mathcal{P})$ to be rationalized by an aggregate ex-post accuracy model.

Sufficiency is the challenge. I assume there is a $p \in \mathcal{P}$ such that $\underline{s}(p) < \overline{s}(p)$, otherwise sufficiency is easy to establish by making learning costly enough (since belief is fixed). It is without loss to assume $\mathcal{P} = \{p_1, p_2, \ldots, p_n\}$, with $p_1 < \cdots < p_n$. Notice that it is also without loss to assume that between any two adjacent prices p_i and p_{i+1}, if $\Pr(\mathrm{Y}|p)$ decreases, then only one of $\Pr(\mathrm{Y}|\underline{\omega}, p)$ and $\Pr(\mathrm{Y}|\overline{\omega}, p)$ decreases, since otherwise I can always enrich $\Pr(\mathrm{Y}|\Omega, \mathcal{P})$ in a way so that behavior is still rationalized by a costly learning model, and so that this is true. I can then rationalizing the richer dataset, which in turn rationalizes the less rich dataset. Similarly, I can assume without loss that $\Pr(\mathrm{Y}|p_1) = 1$ and $\Pr(\mathrm{Y}|p_n) = 0$, and that there are at least two pairs of prices between which $\Pr(\mathrm{Y}|\overline{\omega}, p)$ strictly decreases. Notice that the change between p_1 and p_2 is then a reduction in $\Pr(\mathrm{Y}|\underline{\omega}, p)$, while the change between p_{n-1} and p_n is then a reduction in $\Pr(\mathrm{Y}|\overline{\omega}, p)$.

I am now going to start pairing reductions in $\Pr(\mathrm{Y}|\underline{\omega}, p)\mu(\underline{\omega})$ with reductions in $\Pr(\mathrm{Y}|\overline{\omega}, p)\mu(\overline{\omega})$. The goal is to rationalize each pair with a type, using Lemma 2, but I have to be careful about how I construct the pairs, and I actually start off with an initial pairing that intentionally fails (given the first property below), but fails in a way I can fix (given the second, third, and fourth, properties below).

Initially (henceforth initial pairing), pair reductions $\delta_{\underline{\omega}}$ in $\Pr(\mathrm{Y}|\underline{\omega},\, p)\mu(\underline{\omega})$ between two adjacent prices, with reductions $\delta_{\overline{\omega}}$ in $\Pr(\mathrm{Y}|\overline{\omega},\, p)\mu(\overline{\omega})$ between higher adjacent prices that are not p_{n-1} and p_n (none of the reduction in $\Pr(\mathrm{Y}|\overline{\omega},\, p)\mu(\overline{\omega})$ between p_{n-1} and p_n is paired in the initial pairing), so that the following four properties are satisfied.

First, all reduction in $\Pr(\mathrm{Y}|\underline{\omega},\, p)$ is paired, all reduction in $\Pr(\mathrm{Y}|\overline{\omega},\, p)$ except that between p_{n-1} and p_n is paired. Second, each pairing has $\frac{\delta_{\underline{\omega}}}{\mu(\underline{\omega})} > \frac{\delta_{\overline{\omega}}}{\mu(\overline{\omega})}$. Third, the pair that has the strictly lowest $\delta_{\underline{\omega}}/(\delta_{\underline{\omega}} + \delta_{\overline{\omega}})$ is a pairing (denoted $(\tilde{\delta_{\underline{\omega}}},\, \tilde{\delta_{\overline{\omega}}})$) between reduction in $\Pr(\mathrm{Y}|\underline{\omega},\, p)\mu(\underline{\omega})$ between p_1 and p_2, and reduction in $\Pr(\mathrm{Y}|\overline{\omega},\, p)\mu(\overline{\omega})$ between the lowest pair of prices that have a reduction in $\Pr(\mathrm{Y}|\overline{\omega},\, p)\mu(\overline{\omega})$, call them $p_j,\, p_{j+1}$. Fourth, if I pick $u(\overline{\omega}) = p_n$, and pick $u(\underline{\omega}) < p_1$ so that:

$$\frac{\tilde{\delta_{\underline{\omega}}} u(\underline{\omega}) + \tilde{\delta_{\overline{\omega}}} u(\overline{\omega})}{\tilde{\delta_{\underline{\omega}}} + \tilde{\delta_{\overline{\omega}}}} = \frac{\tilde{\delta_{\underline{\omega}}} p_1 + \tilde{\delta_{\overline{\omega}}} p_j}{\tilde{\delta_{\underline{\omega}}} + \tilde{\delta_{\overline{\omega}}}}, \tag{1}$$

then I have:

$$\frac{(\mu(\underline{\omega}) - \tilde{\delta_{\underline{\omega}}}) u(\underline{\omega}) + (\mu(\overline{\omega}) - \tilde{\delta_{\overline{\omega}}}) u(\overline{\omega})}{(\mu(\underline{\omega}) - \tilde{\delta_{\underline{\omega}}}) + (\mu(\overline{\omega}) - \tilde{\delta_{\overline{\omega}}})} < \frac{(\mu(\underline{\omega}) - \tilde{\delta_{\underline{\omega}}}) p_2 + (\mu(\overline{\omega}) - \tilde{\delta_{\overline{\omega}}}) p_{j+1}}{(\mu(\underline{\omega}) - \tilde{\delta_{\underline{\omega}}}) + (\mu(\overline{\omega}) - \tilde{\delta_{\overline{\omega}}})}. \tag{2}$$

Why is it possible to satisfy all four of the properties simultaneously? The second property is possible because of the first property, and ensures that a reduction in $\Pr(\mathrm{Y}|\underline{\omega},\, p)$ is always paired with a smaller reduction in $\Pr(\mathrm{Y}|\overline{\omega},\, p)$. When I pick the pair with the strictly lowest $\delta_{\underline{\omega}}/(\delta_{\underline{\omega}} + \delta_{\overline{\omega}})$ in the third property, I can make sure the reduction in $\Pr(\mathrm{Y}|\underline{\omega},\, p)$ is paired with an arbitrarily close but smaller reduction in $\Pr(\mathrm{Y}|\overline{\omega},\, p)$. This makes the two values:

$$\frac{\tilde{\delta_{\underline{\omega}}} u(\underline{\omega}) + \tilde{\delta_{\overline{\omega}}} u(\overline{\omega})}{\tilde{\delta_{\underline{\omega}}} + \tilde{\delta_{\overline{\omega}}}} \text{ and } \frac{(\mu(\underline{\omega}) - \tilde{\delta_{\underline{\omega}}}) u(\underline{\omega}) + (\mu(\overline{\omega}) - \tilde{\delta_{\overline{\omega}}}) u(\overline{\omega})}{(\mu(\underline{\omega}) - \tilde{\delta_{\underline{\omega}}}) + (\mu(\overline{\omega}) - \tilde{\delta_{\overline{\omega}}})}$$

arbitrarily close together, which ensures the fourth property can be satisfied since satisfying (1) and continuity then implies (2).

The behavior of the $(\tilde{\delta_{\underline{\omega}}},\, \tilde{\delta_{\overline{\omega}}})$ pairing can be rationalized by a single type according to Lemma 2, and this continues to be true as long as I spread $u(\underline{\omega})$ and $u(\overline{\omega})$ away from each other so (1) is satisfied. Further, the second property then implies such spreads that maintain the equality in (1) increase the mean $m = \mu(\underline{\omega})u(\underline{\omega}) + \mu(\overline{\omega})u(\overline{\omega})$.

Then, if I look at how the ratios change when I change the weight placed on the lower outcome, I see:

$$\frac{\partial\Big(\dfrac{\delta_{\underline{\omega}}u(\underline{\omega})+\delta_{\overline{\omega}}u(\overline{\omega})}{\delta_{\underline{\omega}}+\delta_{\overline{\omega}}}\Big)}{\partial\delta_{\underline{\omega}}}=\frac{\delta_{\overline{\omega}}(u(\underline{\omega})-u(\overline{\omega}))}{(\delta_{\underline{\omega}}+\delta_{\overline{\omega}})^2}<\frac{\delta_{\overline{\omega}}(p_{i+1}-p_{k+1})}{(\delta_{\underline{\omega}}+\delta_{\overline{\omega}})^2}=\frac{\partial\Big(\dfrac{\delta_{\underline{\omega}}p_{i+1}+\delta_{\overline{\omega}}p_{k+1}}{\delta_{\underline{\omega}}+\delta_{\overline{\omega}}}\Big)}{\partial\delta_{\underline{\omega}}}. \tag{3}$$

So, for all other pairings, where $\delta_{\underline{\omega}}$ is from between p_i and p_{i+1}, and $\delta_{\overline{\omega}}$ is from between p_k and p_{k+1}, I have:

$$\frac{\delta_{\underline{\omega}}u(\underline{\omega})+\delta_{\overline{\omega}}u(\overline{\omega})}{\delta_{\underline{\omega}}+\delta_{\overline{\omega}}}<\frac{\delta_{\underline{\omega}}p_{i+1}+\delta_{\overline{\omega}}p_{k+1}}{\delta_{\underline{\omega}}+\delta_{\overline{\omega}}}.$$

What I do next is I reduce each $\delta_{\underline{\omega}}$ to $\hat{\delta_{\underline{\omega}}}$ (leaving the excess unpaired for now), so that:

$$\frac{\hat{\delta_{\underline{\omega}}}u(\underline{\omega})+\delta_{\overline{\omega}}u(\overline{\omega})}{\hat{\delta_{\underline{\omega}}}+\delta_{\overline{\omega}}}=\frac{\hat{\delta_{\underline{\omega}}}p_{i+1}+\delta_{\overline{\omega}}p_{k+1}}{\hat{\delta_{\underline{\omega}}}+\delta_{\overline{\omega}}}\geq\frac{\hat{\delta_{\underline{\omega}}}p_2+\delta_{\overline{\omega}}p_{j+1}}{\hat{\delta_{\underline{\omega}}}+\delta_{\overline{\omega}}}>\frac{\hat{\delta_{\underline{\omega}}}p_1+\delta_{\overline{\omega}}p_j}{\hat{\delta_{\underline{\omega}}}+\delta_{\overline{\omega}}}. \tag{4}$$

Then (3) tells us there is a unique $\hat{\delta_{\underline{\omega}}}$ that satisfies the equality in (4). Further, the last inequality from (4) implies that for each resultant pairing:

$$\frac{\hat{\delta_{\underline{\omega}}}}{\hat{\delta_{\underline{\omega}}}+\delta_{\overline{\omega}}}<\frac{\tilde{\delta_{\underline{\omega}}}}{\tilde{\delta_{\underline{\omega}}}+\tilde{\delta_{\overline{\omega}}}},\text{ and }\frac{\hat{\delta_{\underline{\omega}}}}{\hat{\delta_{\underline{\omega}}}+\delta_{\overline{\omega}}}<\frac{\mu(\underline{\omega})-\tilde{\delta_{\underline{\omega}}}}{(\mu(\underline{\omega})-\tilde{\delta_{\underline{\omega}}})+(\mu(\overline{\omega})-\tilde{\delta_{\overline{\omega}}})}. \tag{5}$$

This implies:

$$\frac{\hat{\delta_{\underline{\omega}}}u(\underline{\omega})+\delta_{\overline{\omega}}u(\overline{\omega})}{\hat{\delta_{\underline{\omega}}}+\delta_{\overline{\omega}}}=\frac{\hat{\delta_{\underline{\omega}}}p_{i+1}+\delta_{\overline{\omega}}p_{k+1}}{\hat{\delta_{\underline{\omega}}}+\delta_{\overline{\omega}}}\geq\frac{\hat{\delta_{\underline{\omega}}}p_2+\delta_{\overline{\omega}}p_{j+1}}{\hat{\delta_{\underline{\omega}}}+\delta_{\overline{\omega}}}$$

$$>\frac{(\mu(\underline{\omega})-\tilde{\delta_{\underline{\omega}}})p_2+(\mu(\overline{\omega})-\tilde{\delta_{\overline{\omega}}})p_{j+1}}{(\mu(\underline{\omega})-\tilde{\delta_{\underline{\omega}}})+(\mu(\overline{\omega})-\tilde{\delta_{\overline{\omega}}})}>\frac{(\mu(\underline{\omega})-\tilde{\delta_{\underline{\omega}}})u(\underline{\omega})+(\mu(\overline{\omega})-\tilde{\delta_{\overline{\omega}}})u(\overline{\omega})}{(\mu(\underline{\omega})-\tilde{\delta_{\underline{\omega}}})+(\mu(\overline{\omega})-\tilde{\delta_{\overline{\omega}}})},$$

which means that each pair $(\hat{\delta_{\underline{\omega}}}, \delta_{\overline{\omega}})$, not including $(\tilde{\delta_{\underline{\omega}}}, \tilde{\delta_{\overline{\omega}}})$, has a higher mean than the mean that is left after removing only $(\tilde{\delta_{\underline{\omega}}}, \tilde{\delta_{\overline{\omega}}})$, so the pairings with $\hat{\delta_{\underline{\omega}}}$'s have reduced the mean of what is left over.

But, I need to match the reductions in $\Pr(\mathrm{Y}|\underline{\omega}, p)\mu(\underline{\omega})$ that are now unpaired with the reduction in $\Pr(\mathrm{Y}|\overline{\omega}, p)\mu(\overline{\omega})$ between p_{n-1} and p_n. So, beginning with the lowest pair of prices with unmatched change and working my way up I take all of the unmatched reduction in $\Pr(\mathrm{Y}|\underline{\omega}, p)\mu(\underline{\omega})$ between p_m and p_{m+1}, denoted $\bar{\delta_{\underline{\omega}}}$, and match it with enough reduction in $\Pr(\mathrm{Y}|\overline{\omega}, p)\mu(\overline{\omega})$ from between p_{n-1} and p_n, denoted $\bar{\delta_{\overline{\omega}}}$, so that:

$$\frac{\bar{\delta_{\underline{\omega}}}u(\underline{\omega})+\bar{\delta_{\overline{\omega}}}u(\overline{\omega})}{\bar{\delta_{\underline{\omega}}}+\bar{\delta_{\overline{\omega}}}}=\frac{\bar{\delta_{\underline{\omega}}}p_{m+1}+\bar{\delta_{\overline{\omega}}}p_n}{\bar{\delta_{\underline{\omega}}}+\bar{\delta_{\overline{\omega}}}}.$$

Notice:

$$\frac{\partial\left(\dfrac{\delta_{\underline{\omega}}u(\underline{\omega})+\delta_{\overline{\omega}}u(\overline{\omega})}{\delta_{\underline{\omega}}+\delta_{\overline{\omega}}}\right)}{\partial\delta_{\overline{\omega}}}=\frac{\delta_{\underline{\omega}}(u(\overline{\omega})-u(\underline{\omega}))}{(\delta_{\underline{\omega}}+\delta_{\overline{\omega}})^2}>\frac{\delta_{\underline{\omega}}(p_n-p_{m+1})}{(\delta_{\underline{\omega}}+\delta_{\overline{\omega}})^2}=\frac{\partial\left(\dfrac{\delta_{\underline{\omega}}p_{m+1}+\delta_{\overline{\omega}}p_n}{\delta_{\underline{\omega}}+\delta_{\overline{\omega}}}\right)}{\partial\delta_{\overline{\omega}}}, \tag{6}$$

so there is a unique $\bar{\delta}_{\overline{\omega}}$ for each $\bar{\delta}_{\underline{\omega}}$. But,

$$\frac{\bar{\delta}_{\underline{\omega}}p_{m+1}+\bar{\delta}_{\overline{\omega}}p_n}{\bar{\delta}_{\underline{\omega}}+\bar{\delta}_{\overline{\omega}}}>\frac{\bar{\delta}_{\underline{\omega}}p_2+\bar{\delta}_{\overline{\omega}}p_{j+1}}{\bar{\delta}_{\underline{\omega}}+\bar{\delta}_{\overline{\omega}}}>\frac{(\mu(\underline{\omega})-\tilde{\delta}_{\underline{\omega}})p_2+(\mu(\overline{\omega})-\tilde{\delta}_{\overline{\omega}})p_{j+1}}{(\mu(\underline{\omega})-\tilde{\delta}_{\underline{\omega}})+(\mu(\overline{\omega})-\tilde{\delta}_{\overline{\omega}})}>\frac{(\mu(\underline{\omega})-\tilde{\delta}_{\underline{\omega}})u(\underline{\omega})+(\mu(\overline{\omega})-\tilde{\delta}_{\overline{\omega}})u(\overline{\omega})}{(\mu(\underline{\omega})-\tilde{\delta}_{\underline{\omega}})+(\mu(\overline{\omega})-\tilde{\delta}_{\overline{\omega}})},$$

where the second inequality is true due to (3) or (6), which in a fashion similar to how (4) implies (5), implies:

$$\frac{\bar{\delta}_{\underline{\omega}}}{\bar{\delta}_{\underline{\omega}}+\bar{\delta}_{\overline{\omega}}}<\frac{\tilde{\delta}_{\underline{\omega}}}{\tilde{\delta}_{\underline{\omega}}+\tilde{\delta}_{\overline{\omega}}},\ \text{and}\ \frac{\bar{\delta}_{\underline{\omega}}}{\bar{\delta}_{\underline{\omega}}+\bar{\delta}_{\overline{\omega}}}<\frac{(\mu(\underline{\omega})-\tilde{\delta}_{\underline{\omega}})}{(\mu(\underline{\omega})-\tilde{\delta}_{\underline{\omega}})+(\mu(\overline{\omega})-\tilde{\delta}_{\overline{\omega}})}. \tag{7}$$

This all means I am doomed to failure since the mean required by each remaining pair is strictly higher than the mean of the unmatched reductions I have left and I run out of reduction in $\Pr(\mathrm{Y}|\overline{\omega},\, p)\mu(\overline{\omega})$ from between p_{n-1} and p_n before I have paired all the unpaired reduction in $\Pr(\mathrm{Y}|\underline{\omega},\, p)\mu(\underline{\omega})$.

So, how do I do better? I return to the initial pairing, but increase $u(\overline{\omega})$ and decrease $u(\underline{\omega})$ so that (1) is still satisfied, which increases the mean of what is left over after the $(\tilde{\delta}_{\underline{\omega}},\, \tilde{\delta}_{\overline{\omega}})$ pairing. Then, (5) tells us that the $\hat{\delta}_{\underline{\omega}}$ I had previously picked to satisfy the equality in (4) were too low, so this iteration I reduce each $\delta_{\underline{\omega}}$ less (but still reduce given how I picked $(\tilde{\delta}_{\underline{\omega}},\, \tilde{\delta}_{\overline{\omega}})$ to satisfy the second property), which means I have less unmatched reduction in $\Pr(\mathrm{Y}|\underline{\omega},\, p)\mu(\underline{\omega})$ to pair after producing the $\hat{\delta}_{\underline{\omega}}$'s. Further, for any amount of unmatched $\Pr(\mathrm{Y}|\underline{\omega},\, p)\mu(\underline{\omega})$, the previous amount of reduction in $\Pr(\mathrm{Y}|\overline{\omega},\, p)\mu(\overline{\omega})$ from between p_{n-1} and p_n I would have matched it with is too large given (7), since I spread $u(\underline{\omega})$ and $u(\overline{\omega})$ but (1) is still satisfied, so the unmatched $\Pr(\mathrm{Y}|\overline{\omega},\, p)\mu(\overline{\omega})$ goes farther, and eventually this strategy is a success. ■

Proof of Proposition 6. When I show sufficiency in the proof of Proposition 5, I employ Lemma 2, and use agents that always select $s \in \{(0,\, 0),\, (0,\, 1),\, (1,\, 1)\}$. If I use the posterior separable cost functions from the proof of Proposition 5 to fix the cost of $s = (0,\, 1)$, and then use this fixed cost to define an ex-ante accuracy cost function that is the maximal ex-ante cost function for information outcomes that assigns the fixed cost of $s = (0,\, 1)$, I establish sufficiency.

Proof of Proposition 8.

In a decision problem, instead of thinking of an agent's chances of selecting X and Y, and the chances of $\underline{\omega}$ and $\overline{\omega}$, I can think of the agent's chances of selecting a default option (what they would select if they did not learn) and a variant option (not the default option), and the chances of the more likely value $\overline{o}$, and the less likely value $\underline{o}$. Further, if the expected value is not the same as the price ($\mu(\underline{\omega})u(\underline{\omega}) + \mu(\overline{\omega})u(\overline{\omega}) \neq p$), and $\mu(\underline{\omega}) \neq \mu(\overline{\omega})$, then the default and variant, and the more likely and less likely value, are each unambiguously defined.

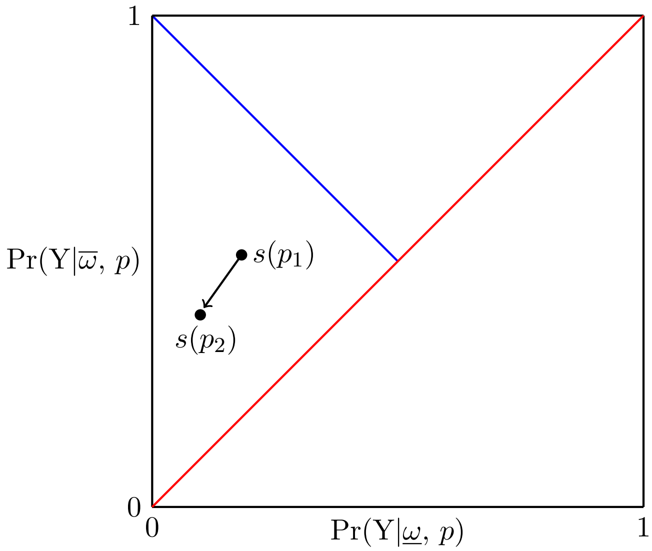
Further, if the cost functions for information outcomes are symmetric (see statement of proposition), if I am given two prior beliefs that are symmetrically located on either side of $\frac{1}{2}$ (see statement of proposition) I can determine the cost of the agent's information outcome if I know the chance they select the variant given the more likely and less likely value. This is true even if I do not know their specific prior, only the symmetric pair of priors it is a member of.

Notice that the assumptions in the statement imply $(u(\underline{\omega}) + u(\overline{\omega}))/2 = p$. Let the difference between p and the realized values be denoted $d \equiv u(\overline{\omega}) - p = p - u(\underline{\omega})$. This means if I normalize the value of the default option to zero, then if the value is $\overline{o}$, the payoff from the variant is $-d$, and if the value is $\underline{o}$, the payoff from the variant is d.

I can solve the resultant optimization problem without knowing what the prior of the agent is. Say I find an optimal pair of probabilities that they select the variant given the more likely and less likely value. Then if their prior is $\underline{\mu}$, the variant is option Y, and the chance they select the variant given the more likely value is an optimal $\underline{s}$ given their prior, and the chance they select the variant given the less likely value is an optimal $\overline{s}$ given their prior (optimal given they are paired with each other).

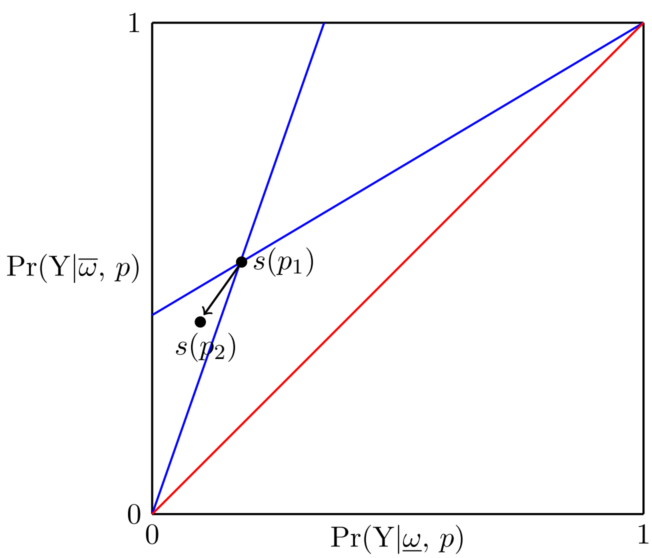
If instead their prior (over the more and less likely values) is $\overline{\mu}$, then the variant is option X, and I know the same probability of selecting the variant given the more likely and less likely value is optimal still optimal given the preceding argument, which tells us selecting option Y when the realized value is $u(\underline{\omega})$ (one minus the probability of selecting the variant in the less likely state) with probability $1-\overline{s}$ is optimal, and selecting option Y when the realized value is $u(\overline{\omega})$ (one minus the probability of selecting the variant in the more likely state) with probability $1 - \underline{s}$ is optimal (optimal given the agent's prior and that they are paired with each other). The same argument can of course be made in the other direction. ■

Figure 14: Aggregate Behavior: Big Dot Red ($p_1 = 0.25 < p_2 = 0.5$)



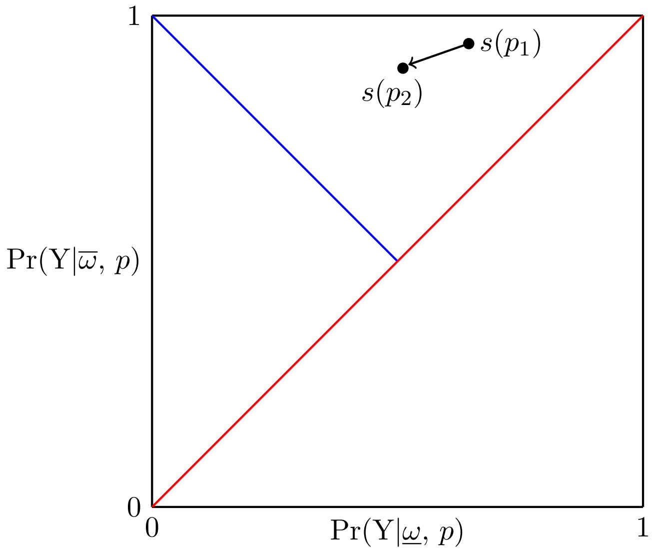
Depicts the behavior from Table 1.
Contradicts the predictions of Proposition 2 (ex-ante model) because there are two $s(p)$ on the same side of blue line and neither is (0, 0) or (1, 1) (deviation significant at 1% level).
See description of Figure 2 for explanation.

Figure 15: Aggregate Behavior: Big Dot Red ($p_1 = 0.25 < p_2 = 0.5$)



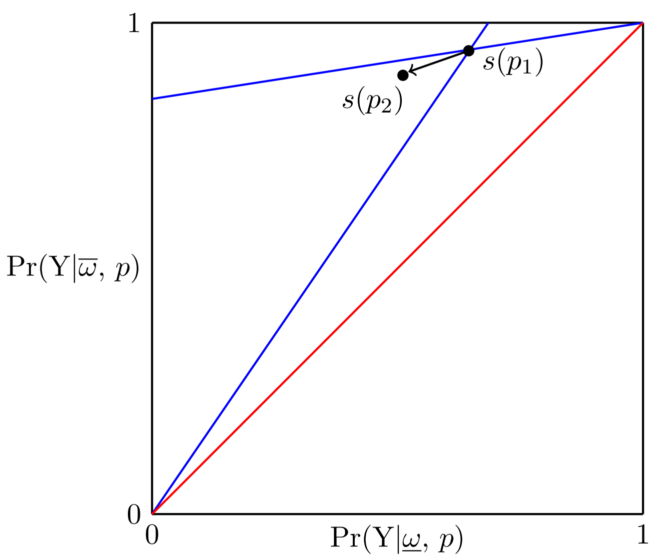
Depicts the behavior from Table 1.
Consistent with the predictions of Proposition 4 (ex-post model) because $s(p)$ moves down and to the left between the blue lines when p increases.
See description of Figure 3 for explanation.

Figure 16: Aggregate Behavior: Big Dot Green ($p_1 = 0.25 < p_2 = 0.5$)



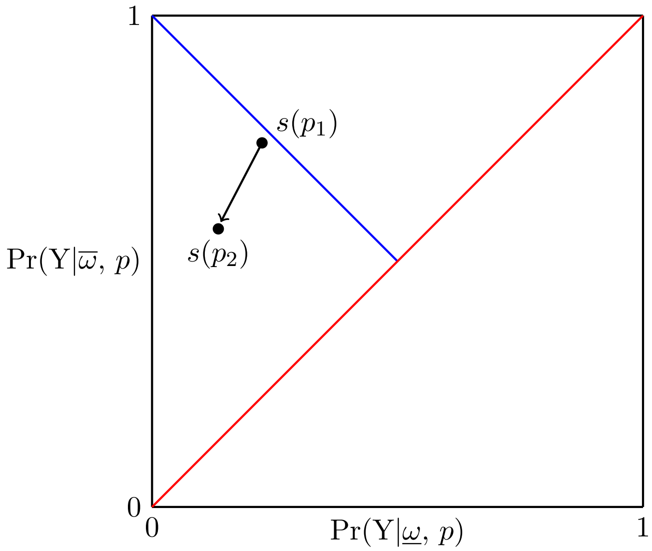
Depicts the behavior from Table 2.
Contradicts the predictions of Proposition 2 (ex-ante model) because there are two $s(p)$ on the same side of blue line and neither is (0, 0) or (1, 1) (deviation significant at 1% level).
See description of Figure 2 for explanation.

Figure 17: Aggregate Behavior: Big Dot Green ($p_1 = 0.25 < p_2 = 0.5$)



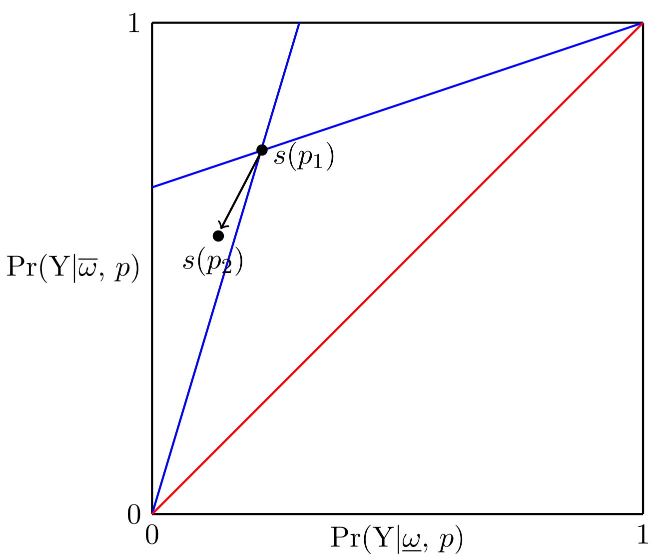
Depicts the behavior from Table 2.
Consistent with the predictions of Proposition 4 (ex-post model) because $s(p)$ moves down and to the left between the blue lines when p increases.
See description of Figure 3 for explanation.

Figure 18: Aggregate Behavior: ($p_1 = 0.25 < p_2 = 0.5$)



Depicts the behavior from Table 3.
Contradicts the predictions of Proposition 2 (ex-ante model) because there are two $s(p)$ on the same side of blue line and neither is (0, 0) or (1, 1) (deviation significant at 1% level).
See description of Figure 2 for explanation.

Figure 19: Aggregate Behavior: ($p_1 = 0.25 < p_2 = 0.5$)



Depicts the behavior from Table 3.
Consistent with the predictions of Proposition 4 (ex-post model) because $s(p)$ moves down and to the left between the blue lines when p increases.
See description of Figure 3 for explanation.